A Theory of Speculation in Community Assets*

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Abstract

We model a community platform where users learn about the quality of its services over time by using its native tokens. The key friction is users can buy tokens for services or trade them primarily for speculation. In the presence of network effects, this tension can lead to situations where no user adopts the platform's services because the risk-adjusted benefit of adoption is lower than that from speculation. Our model can be applied to any asset that derives value from network effects and suggests high token inflation and incentive schemes favoring service usage may be integral to sustaining community participation.

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Alternative asset classes such as digital assets, collectibles, and real estate have become increasingly popular among financial investors.[1] One common theme in these asset classes is that each can be viewed as a community centered around a durable asset, a "community asset", that acts as both a medium for member interactions and a speculative asset that trades in secondary markets. For example, a trading card game community consists of players who both play the game and collect the cards for their price appreciation. On a blockchain-based platform, the asset is a token that is both a means to use the technology and a store of value that can be traded for fiat currencies or other digital assets. Although many members buy these risky assets to enjoy the benefits of these communities, others speculate and primarily hold them for their capital gain. Importantly, this decision to participate or speculate is endogenous to the platform environment. How does speculation in a community asset impact the platform community — especially for new, uncertain platforms that rely on the network effects among their members?

In this paper, we investigate how speculation interacts with adoption when it acts as an outside option to adoption, and users learn about the quality of the platform over time. Our key insight is that speculation may not only hinder learning and adoption, but also lead to a total collapse in participation. This is because the option to speculate acts as an implicit incentive compatibility constraint for adopters that introduces rigidity into each user's adoption decision. In the presence of network effects, a community or platform may thus fall into a situation where no user is willing to use the platform's services at any price. When this happens, the platform's native token has a degenerate price and learning about platform fundamentals ceases even for

[1]For example, a study by Bank of New York Mellon found 41% of the 270+ institutional investors surveyed held digital assets in their portfolios, and 91% expressed interest in investing in them (BONY (2022)). A 2020 survey by Credit Suisse suggests over 70% their ultra high net worth individual clients owned collectibles with an average asset allocation of 5% of their wealth (Credit Suisse (2022)).

platforms with potentially strong fundamentals, giving rise to learning traps. A collapse in participation also paradoxically destroys the option value of tokens that comes from their retradability. This is contrary to the conventional wisdom that speculators provide liquidity and aid in price discovery for risky assets. Our key contribution is to show how price discovery can be distinct from "use" discovery in the presence of a novel negative network externality. In short, speculation can deter the growth of a community.

We model a token platform whose innate benefit depends on an unknown fundamental. A user can buy the platform's tokens in a secondary market and can use its services to receive a noisy benefit. This benefit is amplified by how many other agents use the platform's services at the same time, such that it exhibits network effects (e.g., Katz and Shapiro (1986)). The more users who use the platform's services, the larger the benefit and the stronger it is as a signal about the platform's fundamental. The key friction is a user at any instant can either use the platform's services, and be an "adopter", or hold the tokens primarily for their price appreciation, and be a "speculator".[2] Adopters both buy tokens and incur a heterogeneous flow cost to use the platform's services. They receive both the noisy benefit and the token return. Speculators, in contrast, trade tokens and do not incur the cost. They instead earn only a small fraction of the noisy benefit in addition to the token return.

The key economic mechanism in our model is that the outside option to speculate with a platform's tokens raises the expected return a user requires to become an adopter (i.e., sum of convenience yield and capital gain) above her participation cost. This wedge is largest when uncertainty about the platform fundamental benefit, and consequently token price volatility, is highest. In our continuous-time setting, it is summarized by the Sharpe Ratio from speculating (i.e., the ratio of expected excess return to return volatility). Platform

[2] With conventional stocks and bonds, in contrast, there is a clear distinction between investors in a company and the consumers of its product; as such, speculation is not an outside option for consumers.

participation collapses when the difference in Sharpe Ratios between adopting and speculating is below the participation cost for any potential marginal adopter. This can occur because the Sharpe Ratio of adopting is relatively more exposed to the expected benefit, and current beliefs about the platform's fundamental may be too weak to justify the additional risk from adoption. In contrast, both adopters and speculators share equally in the token's capital gain, which reflects beliefs about the platform's expected *future* benefit.

Our main analysis offers three results. First, we establish that a necessary condition for positive participation is the platform's "adoption information ratio" (the ratio of the expected value of the noisy benefit to its volatility) must be greater than the Sharpe Ratio of speculation when speculators receive only a small fraction of the noisy benefit. The tension that arises between adoption and speculation when users must pay participation costs is essential for platform instability. Without participation costs, similar to the case of owning a stock or bond, or when users can only be adopters and not speculators, there is always an equilibrium with positive participation. Importantly, whether the platform has nonzero participation is not driven by users who will always adopt (i.e., low participation costs) or who will always speculate (i.e., high participation costs), but by users in the middle who face a nontrivial adoption/speculation decision. Second, we show platform instability is increasing in uncertainty about the platform's fundamental. Holding fixed the conditional expectation of the fundamental, increasing uncertainty raises the required return for both adopters and speculators, shrinking the gap between the Sharpe Ratios of both activities. Finally, learning is fastest when the perceived platform fundamental is high because this supports high participation and the revelation of more precise information at each instant. In numerical examples, we demonstrate path-dependence in that positive shocks to beliefs early on can improve the speed of learning, while negative shocks can cause platforms of the same ex ante fundamental to collapse.

We then consider several extensions to explore further the negative network externality introduced by speculation. First, we consider the implications of the conflict between adopters and speculators for optimal platform management. We highlight a novel role for token seigniorage to dampen incentives to speculate on the platform's token by initially inflating their supply. This initial inflation can be beneficial because it eases the tension between adoption and speculation, and can foster an equilibrium with positive participation on the platform. Consequently, while inflation supports adoption, deflation supports speculation. Such a role for token inflation is particularly pronounced for younger platforms with ex ante relatively weak fundamentals. Second, we examine the role of platform incentives that target speculators and adopters, respectively. We show how incentives that favor speculators, such as staking on digital asset platforms, can exacerbate the speculation problem by rewarding speculators for holding unused tokens. Intuitively, this second-use for tokens creates a premium that makes it less attractive for users to adopt the platform's technology. In contrast, incentive schemes that favor adopters, like enthusiasts clubs, can add incentive for users to adopt rather than speculate.

The externalities from speculation we highlight can apply to a variety of community-based asset classes, including digital assets and collectibles. The platform in our model can represent any asset-intermediated community that relies on interactions among users and is subject to uncertainty about its fundamentals. For instance, after accounting for short-sale constraints and indivisibility, our model can help explain housing market phenomena, such as the appearance of "ghost cities" in China where real estate speculation is high but occupancy is low. We focus on digital assets in an extension because speculation is particularly relevant in that asset class.

Our model is related to the literature on community assets like passion assets and collectibles.[3] Bikhchandani et al. (1992) and subsequent work ra-

[3] See Burton and Jacobsen (1999) and Goetzmann et al. (2021) for reviews.

tionalize the rise of fads, such as fashion trends, as information cascades in which agents become sufficiently confident after observing others' actions that they ignore their own private signals and herd. Because agents do not have private information in our setting, our mechanism for adoption is distinct from herding. A tradition following Pesendorfer (1995) views luxury goods, such as high-end fashion, as signaling devices. We instead emphasize the role of such items as retradable assets whose value derives from the social benefits they foster. Communities collapse because of the destabilizing role of speculation with network effects, which differs from crashes in Bulow and Klemperer (1994) where prices drop discontinuously after a buying frenzy because consumers learn from it and delay their purchases. Häckner and Nyberg (1996) considers a model of negative reciprocal externalities where agents value exclusivity to study how high demand induces congestion and reduces willingness to pay. Hughes (2022) estimates the value of rarity, as opposed to scarcity, in a trading card game. Mandel (2009), Lovo and Spaenjers (2018), and Penasse et al. (2021) apply models of conspicuous consumption, emotional dividends with auctions, and heterogeneous beliefs with short-sale constraints, respectively, to explain art markets. In contrast, we focus on how learning and network effects impact the success of community-based assets. Intuitively, we posit if all artwork were hidden in private collections, and access were not subsidized by public venues like museums, artwork would be less valuable.

More recently, speculation has been explored in the context of digital asset platforms. Cong et al. (2021) link the convenience yield to a money-in-the-utility function preference, but do not distinguish between adopters and speculators. Sockin and Xiong (forthcoming, 2023) illustrate how outside speculators can impair token platforms by crowding out and subverting users, which compromises network effects. Mayer (2019) finds speculators provide or take liquidity from adopters depending on the volatility of the platform fundamental. Lee and Parlour (2022) shows how retradability with speculators and heterogeneous beliefs can extract more revenue when launching a project. Athey et al. (2016) shows how speculators in coins like Bitcoin can crowd out users by making returns more sensitive to the capital gain. In our paper, there is no ex ante distinction between users and speculators, and the choice to be one versus the other interacts with network effects and risk premia. Danos et al. (2018) examines a platform where tokens have a cash-in-advance constraint and users either use the service or save tokens at the risk-free rate. There is no learning, uncertainty, or network effects, and consequently no platform instability. In contrast to conventional wisdom in this literature (e.g., Cong et al. (2021), Gryglewicz et al. (2021)), we show token retradability with network effects can hinder rather than accelerate early-stage adoption.

Our paper also contributes to the literature on social experimentation (e.g., Foster and Rosenzweig (1995)) Chamley and Gale (1994) highlights a free-riding effect in which social learning can stall because economic agents do not fully internalize the social benefit of adoption. Persons and Warther (1997) study investment waves in which firms' past adoption of a new technology impacts future adoption through a learning externality because the outcomes of past adoption are publicly observable. Frick and Ishii (2020) study the role of forward-looking consumers who can free-ride off of the information created by other consumer's adoption decisions. Similar to Frick and Ishii (2020), our agents are forward-looking but this is reflected in their hedging demand for tokens rather than the internalization of the free-riding problem. Sákovics and Steiner (2012) explores how a principal optimally subsidizes participation of high and low externality agents, whereas we emphasize that token retradability can cause non-adopting agents to impose negative network externalities on other agents.[4] In contrast to much of this literature, adoption in our setting not only represents a reversible decision, but not adopting also distorts supply

[4]This negative speculation network externality is distinct from many other negative externalities explored in the networks literature, such as congestion externalities like The El Farol Bar problem (e.g., Easley and Kleinberg (2010)).

and potentially crowds out adoption.

Methodologically, our analysis brings convex duality, mean-field game, and Malliavin methods to the study of the economics of community assets. Cong et al. (2022) also formulates a token platform as a mean-field game, but focuses on the dynamics of the wealth distribution from staking, rather than platform stability and network effects. To the best of our knowledge, we are also among the first to provide a computational method for solving for an endogenous cutoff boundary in a mean-field game with network effects.

I. Applications

Consider a community that provides a benefit to agents who use its services. Our paper highlights that speculation can stymie adoption and learning. A community needs three ingredients to create this tension: (1) the community is associated with a tradable asset that allows speculation; (2) its benefits are increasing in the number of agents who adopt, i.e., network effects; and (3) the community's quality is uncertain and participants learn about it over time. In this section, we relate our model to four settings with these properties.

A. Scrip Currencies

A notable example arises in scrip currencies, or credits that can be redeemed for services. Sweeney and Sweeney (1977) recount a story of the Capitol Hill Babysitting Cooperative that issued scrip currency. Co-op members traded scrip with each other in exchange for babysitting duties. However, parents started to hoard scrip and the co-op faced a liquidity trap. This underlying monetary lesson, made infamous by Paul Krugman and other subsequent scholarship,[5] reflects a similar mechanism to the one we present in our model.

B. Collectibles and "Passion" Assets

Our model can also be applied to analyze the economics of collectibles or "passion assets" such as luxury sneakers, classic cars, toys, high-end watches,

[5] See https://slate.com/business/1998/08/baby-sitting-the-economy.html.

baseball cards, and trading cards, amounting to a sizable asset class. Market Decipher estimated the market capitalization of the collectibles market to be $426 billion in 2022. In the context of our model, we can interpret a platform as a supplier of a new community-based collectible of unknown quality that attracts both user interest and speculation.

The underlying collector community and network effects often play key roles in these passion asset markets. For example, and consistent with our theory, the high-end watch market features local communities in the form of watch clubs that facilitate social gatherings for watch enthusiasts and experimental risk-taking for participating brands.[6] Such local clubs are also present among stamp collectors under the American Philatelic Society and informally among shoe collectors known as "sneakerheads." However, asset valuations are driven not only by collector passion and community credibility,[7] but also by speculation. Luxury sneakers, for example, are held by both collectors for nostalgia and status purposes, and investors to speculate on resale websites like Stadium Goods and StockX. In China, speculation became so severe that the People's Bank of China explicitly warned financial institutions against sneaker-trading in October 2019. High-end watch speculation, which picked up heavily in 2017, has also led to backlash from the watch community.[8]

A tension between collectors and speculators is also endemic to the comic book community. From the late 1980s to 1990s, a speculative frenzy overtook the asset class when non-collectors sought to profit from buying new issues and flipping them for profit. Ultimately, and consistent with our theory, this speculative bubble crashed because both established and new publishers satu-

[6]See https://www.gq.com/story/cam-wolf-march-watch-column.

[7]For example, community influence is important in both shoes (https://thesolesupplier.co.uk/news/why-do-we-collect-sneakers/) and watches (https://www.gq.com/story/how-to-start-collecting-watches)

[8]See https://usa.watchpro.com/cooling-prices-could-rid-the-watch-world-of-flippers-and-specula To quote the article, "Everyone from brand to retailer to end user wants to ensure that the watches do not end up in the hands of chancers and flippers."

rated the market with an increased supply of new (and often gimmicky) issues, culminating with the bankruptcy of Marvel in 1997. The importance of network effects in this episode is apparent as publishers dampened enthusiasm among collectors by diluting story quality and increasing character turnover, by "killing off" and creating new characters, to produce "key" books.

As an example of trading card games, consider *Magic: The Gathering*™, which introduces new mechanics over time through new expansions, and players have to play with the cards to discover their usefulness. This community also features adopters, or players who find enjoyment in interacting with others and learning about new game mechanics, and speculators who occupy part of the card supply for pecuniary gain. There is a well-known tension in this community between players and collectors over the reprinting of valuable cards. In contrast, and consistent with our theory, The Pokémon Company printed 9 billion Pokémon cards in 2021 to limit the returns to speculators.[9]

C. Housing Markets

Our insights are also applicable to housing markets. Residents of a neighborhood derive value from their social connections with neighbors. The formation of these communities and the process of evaluating a neighborhood's long-term quality represents a noisy learning process for homeowners (e.g., Gao et al. (2021)). Speculators who buy houses for speculation or short-term rentals, however, can act as a drag on growth in nascent neighborhoods. Bayer et al. (2020), for instance, provide suggestive evidence that housing speculators destabilize housing markets during boom periods, while Gao et al. (2020) show their negative local economic impact during busts. Consistent with our view that speculators do not participate in communities, Gao et al. (2020) measures housing speculation as non-owner occupied home purchases.

The potential for speculation to compromise local network effects in a community is arguably what occurred in Amsterdam after the introduction of

[9] See https://fortune.com/2022/07/06/pokemon-trading-card-shortage-prices-bubble-nine-billion/.

Airbnb (Almagro and Domınguez-Iino (2021)). A similar wave of speculation can explain China's purported "ghost cities," referring to newly constructed cities that have markedly low occupancy rates. This has been partially attributed to speculators buying a substantial fraction of new housing stock, in part at the encouragement of the Chinese government as an investment vehicle for its middle-class. These high-bidding, non-occupying speculators crowd out potential occupants and harm community formation. Such a union of speculation and urbanization is part of a broader push for city growth in Asia and the Arabian Gulf known as "speculative urbanism" (e.g., Goldman (2011)).

D. Digital Assets

Digital asset platforms are salient examples of communities founded on uncertain fundamentals. These platforms distinguish themselves through the specialized services that each provides, such as facilitating transactions, smart contracts, or data-sharing. Like the other communities we discussed, these services improve as the user base grows. A key novelty of these platforms is that they are mediated by a retradable currency, such as utility tokens.[10] The ability to retrade the tokens underpinning these platforms arguably represents an advantage for them over conventional online platforms that use fiat currencies. Beyond providing liquidity, the potential for financial gain acts as a source of enthusiasm for users to experiment with and learn about the quality of these platform's services. However, this financial gain also introduces strong incentives to speculate in these tokens rather than to use them.

Our analysis can help rationalize the sudden collapse of promising platforms, such as the Internet Computer Protocol (ICP), a decentralized internet project with the goal of replacing the existing data-centers. Tremendous enthusiasm surrounded the project, which raised $121M from leading venture capital firms, including Andreessen Horowitz. It premiered as the 7th largest

[10]For instance, Filecoin is a community that allows users to store and retrieve files in a decentralized storage system using FIL tokens. Ethereum enables users to write smart contracts and provides an infrastructure on which to build new services using Ether tokens.

cryptocurrency by market capitalization with a valuation reaching in excess of $70B. Within days of its launch in 2021, however, the ICP token lost over 95% of its value; the platform has yet to fully recover. We plot the history of ICP from inception in Figure 1. Panel A plots the briefly high-flying market capitalization. Panel B plots the number of transactions on the ICP blockchain, compared with the launch of Ethereum in 2015. While ICP launched with tens of thousands of daily user transactions, growth in activity has been lackluster compared to Ethereum, which now hosts over one million transactions per day.

The proximate cause for ICP's collapse is still a puzzle. Interestingly, the ICP token experienced notable price volatility during the initial days of its launch, and outsized speculative activity on exchanges. Our analysis links these observations to the platform's tepid success: the speculative pressure on ICP overwhelmed early adopters and deterred further adoption. Consistent with the importance of experimentation for social learning, ICP tried to jump-start the platform a month after its launch with a "fork" to restart the project.

Our analysis also provides several empirical predictions. First, is that speculation can compete with platform adoption rather than complement it. Second, is that platform usage reveals information about the platform's fundamentals. Third, is that platforms for which there are Decentralized Finance (DeFi) opportunities and that employ the Proof of Stake consensus protocol should see a drag on usage. Consistent with our predictions, Silberholz and Wu (2021) show a roughly 90% decline in token utility usage in the cryptocurrency space relative to speculation since 2017. [11]

II. A Model of a Community Platform

In this section, we present an infinite horizon model of a community platform in which a tradable token is used for the platform's services. There is

[11] Rather striking is the decline in usage is inversely related to the rise of DeFi opportunities. They further provide evidence that higher utility usage by token holders improves price discovery on the platform.

a unit continuum of long-lived users on the platform who at each instant can invest in the token as a speculative vehicle. They also have the option of adopting the platform's technology to benefit from its services. These services can be viewed as the convenience yield from using the token. In what follows, we refer to users who primarily hold the token for its capital gain at time t as speculators, and those who also use the platform's services as adopters. Let π_t be the fraction of users who are adopters at time t and P_t the token price. We assume the platform fundamental A is unknown to all agents.[12]

The service benefit at time t, dD_t, which is the same for all agents, is

$$dD_t = \pi_t A dt + \sigma_D dZ_t^D, \tag{1}$$

where $\sigma_D > 0$ is the diffusion of dD_t and Z_t^D is a standard Wiener process. This captures many salient features that are exhibited by new platforms. Notably, the total benefit from using the platform is noisy. This ensures that users cannot perfectly learn the platform fundamental, A, from observing dD_t, and reflects why many new platforms also launch information campaigns to educate the public. It also represents the instantaneous source of risk from buying the platform's tokens. As in Cong et al. (2021), this risk is not diversifiable for users, and captures systematic fluctuations in the real value of a platform's services. Such shocks could represent shocks to the desirability of platform services or shocks to sentiment that correlate with the broader economy.

Because using a platform's services often relies on having other users on the platform with whom to interact, the benefit from this service exhibits network effects. A user who adopts the platform's services at time t with x_t^a tokens receives an expected benefit A for each token from each other adopting user with whom she interacts. If the size of the community of adopters is π_t,

[12] Assuming a fixed fundamental is convenient for parsimony, and for providing sharp predictions on the platform's life-cycle because learning represents endogenous risk. It is not essential and can be easily relaxed to allow for time-variation to study cyclical behavior.

then she receives a total expected flow benefit of $x_t^a \pi_t A dt$.[13] Importantly, the more users that make use of the platform's services, the larger is this expected benefit, and the more that users learn about this fundamental.

If a user does not adopt the platform's services, she is a speculator. She does not pay a participation cost and instead receives a much smaller fraction $\alpha << 1$ of dD_t as a convenience yield from holding tokens. This smaller yield can be viewed as either a positive externality from adopters or as the revenue from loaning unused tokens.[14]

A. Learning

Since the platform fundamental A is not publicly observable, all users must form expectations about it. We assume that from the perspective of all users at $t = 0$, A is normally distributed $A \sim \mathcal{N}\left(\bar{A}, \sigma_A^2\right)$, and that $\bar{A}$ and σ_A are such that $A > 0$ with probability arbitrarily close to 1. As there is no private information in the economy, and dD_t is the only public signal about A, all users have symmetric information, summarized in the public information filtration $\mathcal{F}_t^c = \sigma\left(\{D_s\}_{s \leq t}\right)$. Because users begin with a normally distributed prior, their beliefs evolve according to the standard Kalman-Bucy filter. In this situation, their posterior for A is Gaussian, $A \mid \mathcal{F}_t^c \sim \mathcal{N}\left(\hat{A}_t, \Sigma_t\right)$ where $\hat{A}_t$ and Σ_t have laws of motion

$$d\hat{A}_t = \frac{\pi_t \Sigma_t}{\sigma_D} d\hat{Z}_t^A \tag{2}$$

$$\frac{d\Sigma_t}{dt} = -\left(\frac{\pi_t \Sigma_t}{\sigma_D}\right)^2, \tag{3}$$

and

$$d\hat{Z}_t^A = \frac{1}{\sigma_D}\left(dD_t - \pi_t \hat{A}_t dt\right), \tag{4}$$

[13] This benefit can reflect collaborative consumption as in Benjaafar et al. (2019).

[14] Speculators receive a small noisy convenience yield to ensure a non-degenerate steady-state when uncertainty dissipates. That it is proportional to dD_t is not necessary. We could assume instead that speculators receive a noisy yield unrelated to it. This would complicate the analysis by introducing market incompleteness, a friction that is not our focus.

is a standard Wiener process with respect to $\mathcal{F}_t^c$ by Girsanov's Theorem. The posterior variance Σ_t can be expressed implicitly as

$$\Sigma_t = \frac{\sigma_A^2}{1 + \left(\frac{\sigma_A}{\sigma_D}\right)^2 \int_0^t \pi_s^2 ds}, \tag{5}$$

and is decreasing over time; as such, $\Sigma_t \in [0, \sigma_A^2]$. This expression also reveals that learning is convex in the size of the adopting population, π_t, and therefore high levels of adoption have a larger impact on learning than a longer time history of low levels of adoption. The fraction of adopters will consequently be the key determinant of how quickly users learn about the platform's fundamental. This is consistent with the intuition that the more users who use the platform, the more public discussion there is about it.

Importantly, equation (1) implies if no user use the platform's services at time t, then $\pi_t = 0$ and users do not learn anything about A in that instant. As a result, their posterior about A, $\mathcal{N}\left(\hat{A}_t, \Sigma_t\right)$, remains unchanged. Given that no users given this posterior, and there is no new information, it follows no users adopt the platform's technology at $t + dt$. Such an outcome can lead to a learning trap on the platform, and contrasts the conventional wisdom that Bayesian learning occurs quickly and within finite time in financial markets.

Because the token price P_t will be adapted to $\mathcal{F}_t^c$, it is an Itô-semimartingale with respect to $\mathcal{F}_t^c$ and we can write

$$dP_t = \mu_{Pt} P_t dt + \sigma_{Pt} P_t d\hat{Z}_t^A, \tag{6}$$

on the region for which there is positive participation.

B. Users

In this subsection, we describe users and the problem that each face. User i for $i \in [0, 1]$ is long-lived and has CARA preferences over her intermediate consumption, c_{it}, such that $u(c) = -e^{-\gamma c}$, and subjective exponential discount rate $\rho > 0$. She can invest in the platform's token and adopt its services, as well as invest in a riskless asset in elastic supply with instantaneous return r. Because users are long-lived and have CARA preferences, they will hedge against the impact that information revelation about the platform fundamental has on future adoption and token prices.

We now describe how user i uses the platform's services. The key friction is a user can either be an adopter or a speculator, and makes this choice at each instant. In both cases, user i will receive the capital gains of their holdings. As an adopter, however, she receives the full service benefit from the platform dD_t; as a speculator, she receives only αdD_t. To adopt the platform's services at time t, user i must pay an instantaneous flow cost, $\pi_t \kappa_i$, where κ_i is innate to her and π_t is the fraction of adopters. These heterogeneous κ_i are drawn from a CDF $G(\kappa)$ with support $[0, \infty)$ and density $g(\kappa)$. This approach succinctly captures many types of users. Users whose κ_i is sufficiently close to 0 will always adopt when $\pi_t > 0$. In contrast, users whose κ_i is sufficiently large will never adopt. In between these two extremes are potential marginal speculators who may adopt depending on the state of the platform. The heterogeneity in participation costs could reflect differences in reservation values, which is often seen as a positive externality in the network literature (e.g., Easley and Kleinberg (2010)). Without wealth effects, this heterogeneity ensures each user will follow a cutoff strategy when deciding whether to adopt.[15]

An adopter incurs her flow participation cost at each instant. This is a common approach in the context of digital assets (e.g., Biais et al. (2023), Cong et al. (2021)), and is sometimes referred to as a per-period fixed cost.[16]

[15] Since CARA utility features second-order risk-aversion, no user would hold zero tokens even when not using the platform's services. As such, the choice between adopting and speculating is without loss.

[16] An alternative cost structure would be to assume users pay a one-time fixed cost and can adopt for free anytime afterward. This approach would strengthen our mechanism because the initial choice to adopt is even riskier when the present-value of all per-period adoption

The functional form of this cost is not qualitatively important as long as it is invariant to token holdings. We conservatively model it as $\pi_t \kappa_i$ so that it is increasing with the fraction of adopters. The scaling by π_t could reflect that transaction costs scale with the number of other adopters with whom she interacts. It further biases toward more adoption on the platform compared to a cost κ_i because $\pi_t \in [0,1]$, and implies a near zero cost for all users near zero adoption (i.e., when $\pi_t \approx 0$). Consequently, our core results on platform instability are even more striking than if we modeled the cost as κ_i.[17]

Let ω_{it} be the fraction of wealth that user i invests in tokens, and a_{it} the indicator for whether the user adopts the platform's services, with $a_{it} = 1$ indicating adoption. The law of motion of user $i's$ wealth W_{it} is

$$dW_{it} = (rW_{it} - c_{it} - \pi_t \kappa_i a_{it})\, dt + \omega_{it} W_{it} \frac{(a_{it} + \alpha\,(1 - a_{it}))\, dD_t + dP_t - rP_t}{P_t}, \tag{7}$$

User i chooses her consumption, investment, and adoption policies to maximize her expected lifetime utility according to the primal problem[18]

$$\begin{aligned} U_0^i &= \sup_{c_i, \omega_i, a_i} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u\,(c_{it})\, dt\right], \\ s.t.: &\quad (7). \end{aligned} \tag{8}$$

It will be convenient to express the primal problem as its convex dual. This will allow us to formulate each user's dynamic programming problem as a HJB-dual variational inequality over the regions in which she is a speculator and in which she is an adopter. For this approach, we solve for optimally

costs must effectively be paid up front (i.e., a fixed cost of $\frac{\kappa}{r}$).

[17]One could also argue for a participation cost $\frac{1}{\pi_t}\kappa_i$, so more users makes it easier to use the platform's services. This approach would further strengthen our channel.

[18]The service benefit here dD_t represents a monetary gain. We could have alternatively modeled the service benefit as a non-pecuniary utility benefit. This approach would introduce wealth effects that would complicate our analysis without providing much additional insight. Online Appendix B shows how this alternative approach leads to similar trade-offs.

invested wealth, $W_{it} = F_{it}$, according to the martingale method of Cox and Huang (1989). Because there is only one source of diffusion risk and two assets, markets are complete. For CARA utility, we can express F_{it} as

$$F_{it} = \frac{1}{r\gamma}\left(\log\gamma + \frac{r-\rho}{r}\right) - \frac{1}{r\gamma}\log\Lambda_{it} + f_i\left(\hat{A}_t, \Sigma_t\right), \tag{9}$$

where Λ_{it} is user i's state price deflator and f_{it} is akin to risk-adjusted wealth.[19] Suppressing time subscripts, let $x_i = \frac{\omega_i F_i}{P}$ be the number of tokens that user i buys and

$$SR\left(a_i\right) = \frac{\left(\alpha + \left(1-\alpha\right)a\right)\pi\hat{A} + \mu_P P - rP}{\left(\alpha + \left(1-\alpha\right)a\right)\sigma_D + \sigma_P P}, \tag{10}$$

the Sharpe Ratio for agent i, which depends on whether the user is an adopter or speculator.

The following proposition characterizes the optimal policies of user i.

Proposition 1 *User i : 1) chooses consumption*

$$c_i = \frac{1}{\gamma}\log\gamma - \frac{1}{\gamma}\log\Lambda_i; \tag{11}$$

2) chooses token demand

$$x_i = \frac{\frac{1}{r\gamma}SR\left(a_i\right) + \partial_{\hat{A}} f_i \frac{\pi\Sigma}{\sigma_D}}{\left(\alpha + \left(1-\alpha\right)a_i\right)\sigma_D + \sigma_P P}, \tag{12}$$

where $a_i = 1$ if she is an adopter and $a_i = 0$ if she is a speculator, respectively, and $f_i\left(\hat{A}, \Sigma\right)$ satisfies the dual HJB variational inequality

$$rf_i = \inf_{a_i}\left\{\pi\kappa_i a_i - \frac{1}{2r\gamma}SR\left(a_i\right)^2 - \partial_{\hat{A}} f_i \frac{\pi\Sigma}{\sigma_D}SR\left(a_i\right)\right\} + \left(\frac{1}{2}\partial_{\hat{A}\hat{A}} f_i - \partial_{\Sigma} f_i\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2; \tag{13}$$

[19] See the proof of Proposition 1 in the Appendix.

with appropriate boundary conditions,[20] *and 3) adopts the platform's services at time t if $\kappa_i \leq \kappa^*$, i.e., $a_i = \mathbf{1}_{\{\kappa_i \leq \kappa^*\}}$, where*

$$\kappa^* = \frac{\left(SR\,(1) + r\gamma \partial_{\hat{A}} f_* \frac{\pi \Sigma}{\sigma_D}\right)^2}{2\pi r\gamma} - \frac{\left(SR\,(0) + r\gamma \partial_{\hat{A}} f_* \frac{\pi \Sigma}{\sigma_D}\right)^2}{2\pi r\gamma}. \tag{14}$$

Proposition 1 characterizes a user's optimal consumption and token demand. Because user i has CARA preferences, her demand for tokens is composed of: 1) a mean-variance term, $x_i^{MV} = \frac{1}{r\gamma} \frac{SR(a_i)}{(a_i + \alpha(1-a_i))\sigma_D + \sigma_P P}$, with effective risk aversion $r\gamma$ because she is long-lived; and 2) a hedging term $x_i^H = \frac{\partial_{\hat{A}} f_i \pi \Sigma / \sigma_D}{(a_i + \alpha(1-a_i))\sigma_D + \sigma_P P}$ that reflects the covariance between token returns and innovations to public beliefs about the platform fundamental.

We can make equation (14) more interpretable by rewriting it as

$$\kappa^* = \frac{r\gamma}{2\pi} \left(Var\left[x_*^a dr_t^* \mid \text{Adopt}\right] - Var\left[x_*^s dr_t^* \mid \text{Do Not Adopt}\right]\right). \tag{15}$$

Whether the marginal user adopts depends on the relative return volatility to holding tokens while adopting, compared to speculating, for that user. When the volatility is too high, then a user will not adopt. As we will see, this outside option of speculating will be a source of rigidity on the platform that can impact learning and exacerbate the fragility of network effects.

C. Equilibrium Definition and Market Clearing

We search for a Markov rational expectations equilibrium in the state variables $\left(\hat{A}, \Sigma\right)$ in which all users choose their optimal policies according to Proposition 1 and the market for tokens clear. We normalize the supply of tokens to unity. Given the agents' cutoff strategy, market clearing requires

$$\int_0^{\kappa^*} x_i^a dG\left(\kappa_i\right) + \int_{\kappa^*}^{\infty} x_i^s dG\left(\kappa_i\right) = 1. \tag{16}$$

[20] We provide a characterization of these boundary conditions in the Online Appendix.

As this is an equilibrium model of a small platform, we do not impose market clearing in the consumption or risk-free asset market.

Because diffusion processes have continuous sample paths, we characterize separately the regions with positive and zero participation, and treat the equilibrium with zero participation as a boundary condition.[21] Finally, if there exist multiple equilibria, we follow the standard practice of assuming the platform coordinates users on the highest participation equilibrium.[22]

III. Equilibrium

In this section, we characterize the token price and the dynamics of adoption on the platform. In what follows, we conjecture that the token price is a function of the two state variables $\left(\hat{A}, \Sigma\right)$, such that by Itô's Lemma

$$\mu_P P = \left(\frac{1}{2}\partial_{\hat{A}\hat{A}} P - \partial_{\Sigma} P\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2, \tag{17}$$

$$\sigma_P P = \partial_{\hat{A}} P \frac{\pi\Sigma}{\sigma_D}. \tag{18}$$

Imposing the optimal investment policies of adopters and speculators and market clearing, we arrive at the following proposition for the token price.

Proposition 2 *The token price satisfies*

$$\begin{aligned} P &= \frac{1}{r}\left(1-(1-\alpha)\,v\frac{1-\pi}{\left(\alpha\sigma_D+\partial_{\hat{A}}P\frac{\pi\Sigma}{\sigma_D}\right)^2}\right)\pi\hat{A}+\frac{1}{r}\left(\frac{1}{2}\partial_{\hat{A}\hat{A}}P-\partial_{\Sigma}P\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 \\ &\quad +\gamma v\int_0^\infty\left(\frac{\mathbf{1}_{\{\kappa_i\leq\kappa^*\}}}{\frac{\sigma_D^2}{\pi\Sigma}+\partial_{\hat{A}}P}+\frac{\mathbf{1}_{\{\kappa_i>\kappa^*\}}}{\frac{\alpha\sigma_D^2}{\pi\Sigma}+\partial_{\hat{A}}P}\right)\partial_{\hat{A}}f_i\,dG\left(\kappa_i\right)-\gamma v, \end{aligned} \tag{19}$$

[21] Although the token price will jump at the zero participation boundary, the value functions and wealth of users will be continuous because they anticipate the potential collapse.

[22] Because a platform profits from seigniorage and transaction fees, the highest participation equilibrium will always maximize its revenue.

where v is the harmonic mean of investors' return variance

$$v^{-1} = \frac{\pi}{\left(\sigma_D + \partial_{\hat{A}} P \frac{\pi \Sigma}{\sigma_D}\right)^2} + \frac{1-\pi}{\left(\alpha\sigma_D + \partial_{\hat{A}} P \frac{\pi \Sigma}{\sigma_D}\right)^2}. \tag{20}$$

A necessary condition for an equilibrium with positive participation to exist when there is uncertainty ($\Sigma_t > 0$) and α sufficiently small is

$$\frac{\pi \hat{A}}{\sigma_D} \geq \left| \frac{\mu_P P - rP}{\sigma_P P} \right|; \tag{21}$$

otherwise, $\pi_t = 0$ and the price collapses to a constant

$$P = -\gamma \left(\alpha \sigma_D\right)^2. \tag{22}$$

Proposition 2 reveals that the token price reflects the motives of both adopters and speculators, weighted by the sizes of the two populations in proportion to the total return variance they bear. Adopters earn a higher expected return from adopting the platform's services but also face a higher risk by being exposed to transient fluctuations in the platform's services. Speculators, in contrast, receive a lower expected return but also bear less risk. The convenience yield ($\pi \hat{A}$) and the hedging and market risk premium, the last two terms in equation (19), will therefore be below that demanded by adopters because speculators do not value the convenience yield and provide liquidity.

In addition, Proposition 2 characterizes a necessary condition for existence on the token platform. We refer to the benefit-to-cost ratio of adoption, or the expected transaction benefit divided by its volatility, $\frac{\pi \hat{A}}{\sigma_D}$, as the "adoption information ratio". This condition requires that the adoption information ratio exceeds the risk-return trade-off (the Sharpe Ratio) for the capital gain,

$\left|\frac{\mu_P P - rP}{\sigma_P P}\right|$.[23] Although both are endogenous objects, the necessary condition is useful for highlighting the relevant trade-off and the key mechanism for determining community participation. The token's capital gain essentially competes with the adoption information ratio, and a sufficient number of users must adopt to induce other users to adopt it as well. If the relative volatility of adoption is high, and consequently the information ratio is too low relative to the Sharpe Ratio, then participation collapses and learning will cease.

A classical benchmark for our analysis is the case in which there are no participation costs to adopt the platform's services (i.e., $\kappa_i = 0$ for all i). In this case, the token price and learning dynamics reduce to the canonical case of a financial asset, such as a stock, that pays a noisy dividend with an unobserved persistent component, A. There is always full participation ($\pi_t = 1$) and learning occurs at the fastest possible rate because of the strong signal from the instantaneous transaction benefit, dD_t, when $\pi_t = 1$. We characterize this benchmark in the following proposition.

Proposition 3 *In the absence of participation costs (i.e., $\kappa_i = 0$ for all i), there is full participation ($\pi_t = 1$) and the token price satisfies*

$$P = \frac{1}{r}\hat{A} + p\left(\Sigma\right) - \gamma\sigma_D^2, \tag{23}$$

where $p\left(\Sigma\right)$ is given by (with $y = -\frac{r\sigma_D^2}{\Sigma}$):

$$p\left(\Sigma\right) = \gamma\sigma_D^2 \exp\left(\frac{r\sigma_D^2}{\Sigma}\right)\int_{-\infty}^{-\frac{r\sigma_D^2}{\Sigma}}\left(\frac{2}{y'} - \frac{1}{y'^2}\right)\exp\left(y'\right)dy', \tag{24}$$

and uncertainty $\Sigma_t = \left(\sigma_A^{-2} + \sigma_D^{-2}t\right)^{-1}$ declines deterministically over time.

Proposition 3 reveals the token price without participation costs is linear

[23] We take the absolute value because speculators short tokens when the expected excess return is negative.

in the perceived fundamental, $\hat{A}$, and its risk premium, $-\gamma\sigma_D^2$, and a nonlinear function of uncertainty, Σ. In this setting, uncertainty decreases deterministically over time because there is no fluctuation in the fraction of adopters. As is apparent from this benchmark, the nontrivial adoption problem faced by users is essential for a nontrivial relation between the token price, learning dynamics, and network effects on the platform.

In the next two subsections, we examine how this adoption problem impacts platform stability and performance. We provide plots from a numerical example of our model to illustrate our results.[24]

A. Platform Stability

In this subsection, we show that the platform only breaks down when users have the option to speculate, not when speculators are either absent or a distinct group of token holders. This potential for breakdown bifurcates the state space of the posterior of the platform fundamental $\mathcal{N}\left(\hat{A}_t, \Sigma_t\right)$ into two regions: one in which there is adoption and the platform survives and one in which the platform fails permanently. This failure boundary can be represented by a critical conditional expectation of the fundamental, $\hat{A}^c\left(\Sigma_t\right)$, that is decreasing in uncertainty about the fundamental, Σ_t, i.e., when there is more uncertainty, users are less willing to adopt on a weaker platform.

An important limiting case that will help us characterize the equilibrium is when there is no uncertainty about the platform fundamental. This case not only helps highlight the impact of uncertainty and learning on the platform, but also represents one of the two steady-state outcomes; the other is that the platform fails and learning stalls. Proposition 4 characterizes the equilibrium.

Proposition 4 *When there is no uncertainty about the platform fundamental,*

[24]Online Appendix C provides details on the numerical procedure.

A (i.e., $\Sigma_t = 0$): 1) if $A \geq A^c$ the token price is

$$P = \frac{\alpha}{r} \frac{1-(1-\alpha)\pi}{1-(1-\alpha^2)\pi} \pi A - \frac{\gamma \alpha^2 \sigma_D^2}{1-(1-\alpha^2)\pi}, \tag{25}$$

user i adopts the platform if

$$\kappa_i \leq \kappa^* = \frac{1}{2\pi r \gamma} \frac{1}{\sigma_D^2} \left(\frac{1}{\alpha} - 1 \right) rP \left(2\pi A - \left(1 + \frac{1}{\alpha} \right) rP \right), \tag{26}$$

and the fraction of users that adopt, π, is increasing in A in the high participation equilibrium;

and 2) if $A < A^c$, learning ceases and no equilibrium with positive participation exists.

Without uncertainty about the platform fundamental, A, there are three possible outcomes: either there is zero participation or there are two equilibria with positive participation, one with high and one with low participation. The former occurs when A is below a critical cutoff A^c. When A is above this cutoff, there is a constant population of users adopts at each instant, with the size of this population increasing in A in the high participation equilibrium, which is our focus. In the latter case, the token price capitalizes a transaction benefit that is a weighted average of adopters and speculators who provide liquidity to adopters. Not all users become adopters because the return to adopting is below the per-period cost of the highest-cost (largest κ_i) users. Interestingly, the platform resembles a "pay-as-you-go" platform in this limit in which the total cost of services at each date is constant.

Such limiting behavior provides insight into how the platform behaves under uncertainty. When there is arbitrarily small uncertainty about the platform fundamental A, then it is clear that far away from the breakdown threshold, A^c, uncertainty has no impact on whether users adopt the platform. Near A^c, however, the additional risk gives rise to a risk premium for the same expected transaction benefit $\pi\hat{A}$ that makes adoption the less attractive option. As such, the breakdown boundary is above A^c. We can repeat these arguments by backward induction because uncertainty only (weakly) falls over time to establish the following corollary.

Corollary 1: There exists an adapted process $\hat{A}^c(\Sigma_t)$ *that is increasing in* Σ_t *such that the platform survives if* $\hat{A}_t \geq \hat{A}^c(\Sigma_t)$ *and breaks down (i.e., no adoption) otherwise. As* $\hat{A}_t$ *and* π_t *increase, speculators take less positive / more negative token positions.*

Corollary 1 reveals the behavior of the platform over time is determined by users' expected value of A, $\hat{A}_t$. When it is high (i.e., higher than $\hat{A}^c(\Sigma_t)$), then there is adoption of the platform's services, and learning continues until all users learn A and the platform converges to its steady-state. If instead $\hat{A}_t$ falls below the platform's critical threshold at some time t, then there is no adoption and learning stalls. As the conditions on the platform are the same at $t + dt$ as at t, the platform also fails at $t + dt$ and consequently all future t. In such a situation, there is a learning trap and the platform breaks down because of common pessimism about the quality of its services.

The second part of the corollary reveals that speculators take positions that are decreasing in the platform fundamental and in how many users adopt. This is because both raise the convenience yield for adopters, which pushes up the token price. As a result, speculators who receive a smaller convenience yield take a smaller long position and even a negative or short position in tokens for $\hat{A}_t$ and π_t sufficiently large, depending on uncertainty Σ_t. In the limit of no uncertainty ($\Sigma_t = 0$), speculators are always shorting tokens and subsidizing adopter participation because there is no capital gain from holding them.

We illustrate the relation between uncertainty and the boundary of positive participation in Figure 2. To the right of this boundary, the conditional expectation of users about the platform fundamental, $\hat{A}_t$, is sufficiently high given the uncertainty about A, Σ_t, to sustain positive participation. To the left of this boundary, however, $\hat{A}_t$ is too low relative to Σ_t, and participation collapses (i.e., $\pi_t = 0$). This boundary, $\hat{A}^c(\Sigma_t)$, is increasing in uncertainty, confirming that when Σ_t is high (i.e., on younger platforms), more optimistic beliefs among users (i.e., a higher $\hat{A}_t$) are required to sustain participation.

A key question in this paper is what role does the outside option to speculate play in platform behavior and in the speed of learning. An important benchmark is the special case in which a user must adopt the platform to buy tokens, i.e., she cannot speculate. This corresponds to the conventional approach to modeling users in the digital asset literature, in which there may be a separate group of speculators who never adopt the platform. The following proposition establishes the behavior of the platform in this special case.

Proposition 5 *When users can only adopt the platform services: 1) when there is no uncertainty about the platform fundamental, A, (i.e., $\Sigma_t = 0$), the token price is*

$$P = \frac{1}{r}\pi A - \frac{\gamma}{\pi}\sigma_D^2, \tag{27}$$

the fraction of users that adopt, π, is positive and constant, and π and the transaction benefit (πA) are ex ante more volatile with speculators;

2) with uncertainty (i.e., $\Sigma_t > 0$), there is always an equilibrium with $\pi_t > 0$ where users learn the true fundamental arbitrarily well in finite time.

3) There is positive participation even if there is a separate group of speculators on the platform.

Proposition 5 highlights several important observations. First, when users cannot be speculators, there is always an equilibrium with positive participation. This is because the price can always fall enough to support a positive mass of adopters by providing a positive excess return to the marginal adopter. In contrast, when users can speculate the platform can collapse because being a speculator acts as an outside option to adopting, which dampens participation. Therefore, it is not because of the complementarity of network effects alone the platform may fail, but because of the dual use of the token. Second, because there is always an equilibrium with adoption, learning about the platform fundamental occurs at each instant, and users consequently always learn its true value in the long-run. Third, the presence of outside speculators who are distinct from users does not compromise stability on the platform. This highlights that it is the role of speculation as an outside option, and not as a source of token demand, that destabilizes community asset platforms.

B. Platform Performance

In this subsection, we illustrate the performance of the platform using our numerical example. Figure 3 illustrates how the rigidity introduced by the option to be a speculator impacts platform performance and token price. It relates the fraction of users that adopt the platform's services π as a function of their conditional expectation of the platform fundamental, $\hat{A}_t$ for different levels of uncertainty, Σ_t. Higher uncertainty results in lower participation and a larger region of conditional beliefs for which there is no participation.

Figure 4 plots simulations of our model starting from a common prior belief, $\mathcal{N}(\hat{A}_0, \Sigma_0)$. Panel A plots the time-series of the conditional expectation of $A, \hat{A}_t$, Panel B the conditional uncertainty about A, Σ_t, and Panel C the fraction of adopters π_t over time. In simulations where $\hat{A}_t$ drifts higher, users adopt at a higher rate and learn faster, resulting in a lower Σ_t over time. If beliefs $(\hat{A}_t, \Sigma_t)$ fall to the boundary in Figure 2, $\hat{A}^c(\Sigma_t)$, then a bad shock to $\hat{A}_t$ can push beliefs into the region of no participation where $\pi_t = 0$. This occurs in the "flat-lined" sample learning paths. In this case, all future learning ceases and there is no further updating of beliefs.

More generally, speculation enhances the performance on stronger platforms (high $\hat{A}_t$) and harms it on ex ante weaker platforms (low $\bar{A}$). When the expectation of the fundamental is relatively high, participation is high, learning is fast, and speculators short tokens to provide liquidity to adopters. The opposite happens when the perceived fundamental is relatively low.

IV. Extensions

In this section, we consider several extensions of our baseline model that are relevant for current platform design decisions. First, we consider the optimal seigniorage policy of the platform. We then analyze how incentive schemes targeted at speculators and adopters, respectively, impact adoption.

A. Platform Design

In this subsection, we explore the implications of the tension between adopters and speculators for platform design. Our key insight is a platform owner who internalizes this tension will want to issue enough tokens initially to ensure there is positive adoption by dampening the returns to speculation.

We consider the problem of a token platform run by an owner who profits from seigniorage and cannot commit to an inflation schedule. At a given instant t, the supply of tokens on the platform is M_t, and the owner can issue or buy tokens at the market-clearing token price. Issuing tokens causes inflation on the platform, however, which dilutes the claims of users over time. To reflect this, the noisy service benefit is divided over the total token supply, such that a user with one token at time t receives dD_t/M_t. With this specification, there is money neutrality regarding real outcomes on the platform because seigniorage does not, in itself, increase the total service benefit to users.

The owner chooses an initial supply M_0 and the inflation schedule on the platform. We consider a smooth (locally deterministic) issuance strategies m_t such that the token supply evolves according to

$$dM_t = m_t dt. \tag{28}$$

The optimal smooth issuance policy will also imply that the owner will not make discrete issuances, ΔM_t. The owner discounts profits at the riskless rate r and chooses M_0 and $\{m_t\}_{t\geq 0}$ to maximize the present value of its profits

$$V_0 = \sup_{M_0, m_t} P_0 M_0 + \Pi_0, \tag{29}$$

where

$$\Pi\left(\hat{A}_t, \Sigma_t, M_t\right) = \mathbb{E}\left[\int_t^\infty e^{-r(s-t)} P_s m_s ds\right], \tag{30}$$

is the owner's continuation profits and $\mathcal{N}(\hat{A}_t, \Sigma_t)$ the posterior belief about A. The following proposition characterizes the optimal seigniorage policy.[25]

Proposition 6 *The platform owner: 1) chooses the initial token issuance M_0 such that there is positive participation on the platform and*

$$M_0 = -\frac{P_0 + \partial_M \Pi\left(\bar{A}, \sigma_A^2, M_0\right)}{\partial_M P_0}; \tag{31}$$

2) chooses a continuation issuance policy m such that

$$P = -\partial_M \Pi; \tag{32}$$

and 3) manipulates the token price such that expected excess return yield

$$\left(\mu_P - r\right) P = 2r\Pi\partial_M \pi / \pi. \tag{33}$$

The platform owner chooses the initial token issuance to ensure positive participation, and then continues to issue tokens to maintain it. She issues tokens at each instant until the marginal benefit, the token price P, equals the marginal cost, the marginal loss in future seigniorage revenue, which can involve issuing or buying back tokens. She has this incentive because if participation collapses, the token has zero (or even negative) value and she cannot profit from future seigniorage without incurring the cost of issuing tokens at a loss to restore participation. This optimal issuance policy further manipulates the capital gains from holding tokens by linking their expected excess return, $\left(\mu_P - r\right) P$, to her continuation profits and the marginal impact of increasing the token supply on the size of the adopting population, $\partial_M \pi / \pi$. When the

[25] We use variational methods to solve for time-consistent optimal issuance policies.

expected excess return is positive, she wants to increase π to increase the conditional volatility of $\hat{A}_t$, $\left(\frac{\pi\Sigma}{\sigma_D}\right)^2$, to raise the token's price. When it is negative, she wants to lower this volatility to maintain a high token price.

Our analysis clarifies the differential impact of inflation on motives to adopt and speculate on the platform. While inflation supports adoption, deflation supports speculation. Further, we highlight a novel role for token inflation to mitigate the tension between the two. Given such inflation is meant to ensure adoption, this mechanism is pronounced on platforms with ex ante weaker fundamentals (i.e., low $\bar{A}$).[26]

B. Incentives for Speculators

In this subsection, we illustrate how incentives that differentially reward speculators may have unintended consequences for participation and social learning on token platforms. Although any friction that reduces the benefits to joining the platform will hamper adoption (and consequently learning), such as transaction fees levied by a platform owner, subsidies that primarily benefit speculators introduce additional nuanced issues. In the context of passion assets, this could reflect supply restrictions that raise the rarity of certain collectibles or the ability to use such assets as collateral in lending. In the context of digital asset platforms, this could reflect subsidies afforded to the backing the token, such as staking with the Proof of Stake protocol.

We focus on the Proof of Stake example and show it can exacerbate the negative externalities from speculation by allowing speculators to free-ride on the transaction surplus of adopters and collect seigniorage revenue. In our setting, it is natural for speculators to act as the validators because they do not buy tokens for active use. Adopters, in contrast, use their tokens, and cannot lock them in escrow to be validators.[27] Suppose adopters pay a fraction $\phi > 0$ of their transaction benefit as a transaction fee to validators. In

[26]This insight is relevant as digital assets experiment with token supply schedules. While Bitcoin is inflationary, both Ether and Tron switched to deflationary token schedules.

[27]We abstract from frictions in consensus protocols because it is not our focus.

addition, validators receive a fraction ζ of the existing token base, M_t, as new tokens. For simplicity, we assume a negative token position as a speculator entails shorting these validation cash flows. With seigniorage, users receive a service benefit dD_t/M_t to reflect that inflation erodes token value. In what follows, let $X_t^s = \int_{\kappa^*}^{\infty} x_{it}^s dG(\kappa_i)$ be the number of tokens held by speculators, and $q_t = \frac{M_t - X_t^S}{X_t^S}$ be the number of transactions validated by each speculator. Proposition 7 characterizes the optimal policies of users.

Proposition 7 *Under Proof of Stake, user i : 1) chooses the same consumption, token demand, and adoption policies as in Proposition 1, except now the Sharpe Ratio $SR(a_i)$ is*

$$SR(a_i) = \frac{((q\phi + \alpha)(1 - a_i) + (1 - \phi) a_i) \pi \hat{A}/M + \left(\mu_P + \frac{1-a_i}{X_t^S}\zeta M\right) P - rP}{((q\phi + \alpha)(1 - a_i) + (1 - \phi) a_i) \sigma_D/M + \sigma_P P}. \tag{34}$$

The key observation is validation services allow speculators to free-ride off of adopters by giving them part of the adopters' transaction benefit without paying any participation costs. If $1 - \phi \leq q\phi + \alpha$, or $X_t^S \leq \frac{\phi}{1-\alpha} M_t$, then a positive level of participation cannot be sustained because of the severity of the free-rider problem. Similarly, the ability to stake tokens through DeFi opportunities, such as by providing liquidity to an Automated Market Maker (AMM), can exacerbate the tension raised by speculation by providing a return to not using tokens. Intuitively, introducing a second-use for tokens introduce a premium into their price, which crowds out adopters.

C. Incentives for Adopters

We now illustrate how platform arrangements that favor adopters can help prioritize adoption over speculation. In the context of digital assets, this can take the form of Play-to-Earn incentives that reward adopters with an additional capital gain from token seigniorage, and community tokens that provide an additional convenience yield to adopters by fostering community-building and community-generated content.[28] In the context of high-end watches, some brands, like Omega, sponsor clubs to organize gatherings enthusiasts.

To illustrate how such schemes can impact adoption, let $X_t^a = \int_0^{\kappa^*} x_{it}^a dG(\kappa_i)$ be adopter token holdings, ζMP the seigniorage , and $\pi C > 0$ the total monetary value of the targeted at adopters. The Sharpe Ratio for users becomes

$$SR(a_i) = \frac{(\alpha(1-a_i)+a_i)\pi\hat{A}/M + a_i\pi C/M + \left(\mu_P + \frac{a_i}{X^a}\zeta M\right)P - rP}{(\alpha(1-a_i)+a_i)\sigma_D/M + \sigma_P P}. \tag{35}$$

Analogous to Proposition 2, one can show the key necessary condition for an equilibrium with nontrivial participation for α sufficiently small is relaxed relative to equation (21). There is consequently a role for subsidies for adopters to maximize adoption of the platform's services.

V. Conclusion

In this paper, we construct a model of a community platform with network effects whose services are intermediated by a retradable asset. The key insight is the ability to hold tokens as a speculator rather than use the platform's services acts as a friction that can hamper participation, slow down learning, and lead to learning traps in which viable platforms fail prematurely. Our analysis suggests dampening the impact of token retradability, such as through inflation and differentially rewarding adopters, can improve platform performance by limiting the incentives for speculation. This posits a positive aspect to the dramatic decline in cryptocurrency prices since November 2021 in that it may represent a catalyst for future adoption. Our theory can also explain housing market phenomena, such as the under-utilization of "ghost cities" in China, as well as inflation and the setup of local communities in collectibles markets.

[28] On Socios.com, sports fans can buy fan tokens issued by internationally recognized sports teams to receive VIP offerings and engage in the voting process for the teams' crowd-sourced decisions. Interestingly, the CEO of Socios.com argued its fan tokens are to provide fans with entertainment and genuine utility rather than an investment vehicle. See https://www.sportspromedia.com/news/socios-ceo-alexandre-dreyfus-fan-tokens/.

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Figures

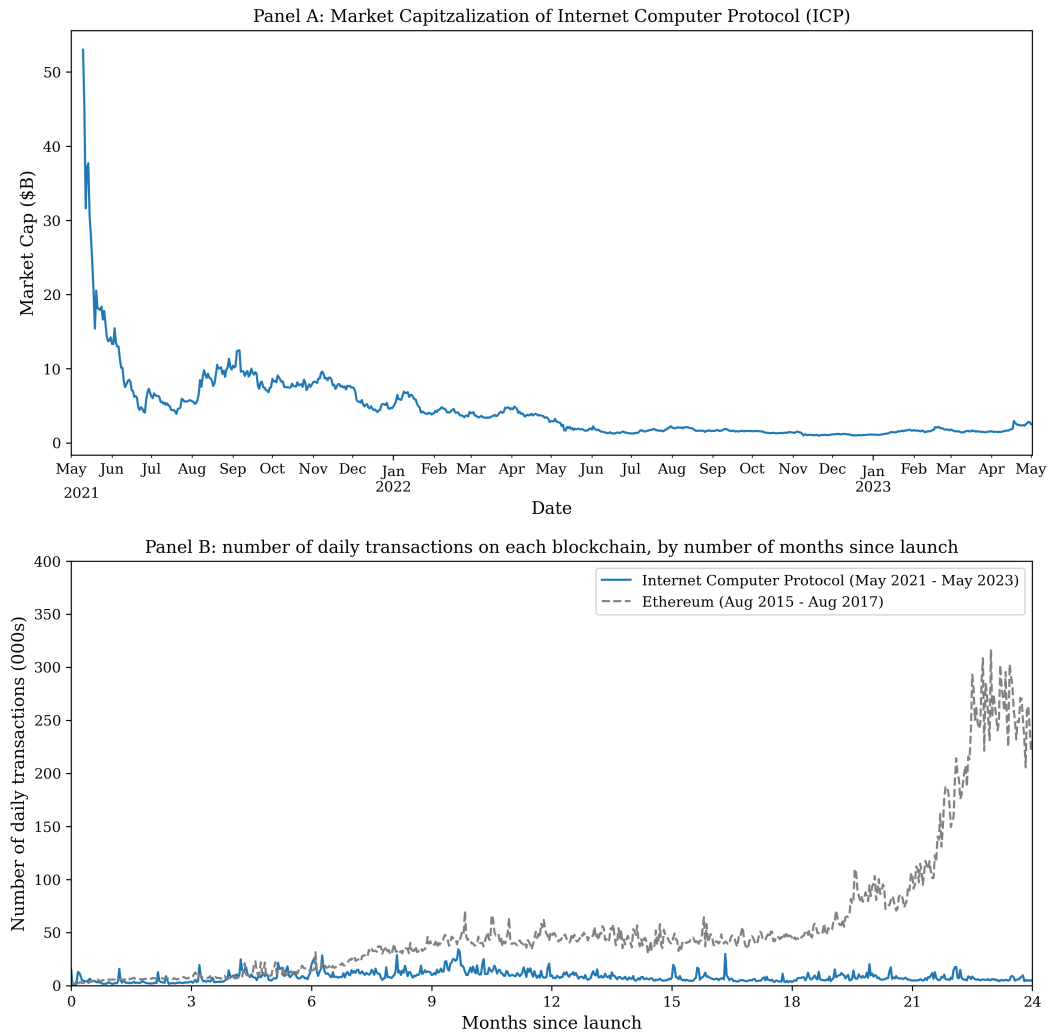

Figure 1: Internet Computer Protocol (ICP) market capitalization and number of transactions since launch in May 2021. Panel A data comes from CoinMarketCap.com. Panel B data on ICP and Ethereum blockchain transactions were downloaded from the ICP website dashboard and Ethereum blockchain API, respectively.

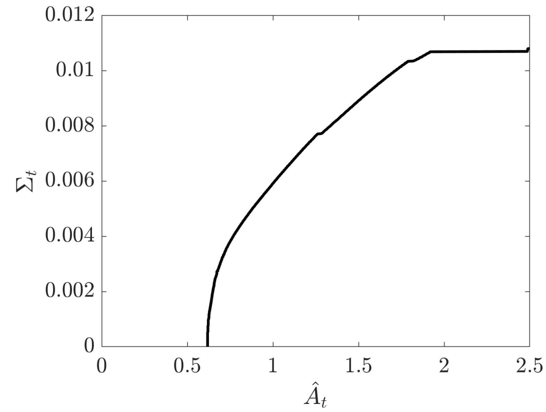

Figure 2: Boundary of positive participation $\hat{A}^c$, as a function of A_t and Σ_t. The parameters for the plot are $r = 0.10,\ \gamma = \frac{1}{r} = 10,\ \sigma_D = 0.5,\ \alpha = 1 \times 10^{-2},\ \log \kappa_i \sim \mathcal{N}(0, 2)$.

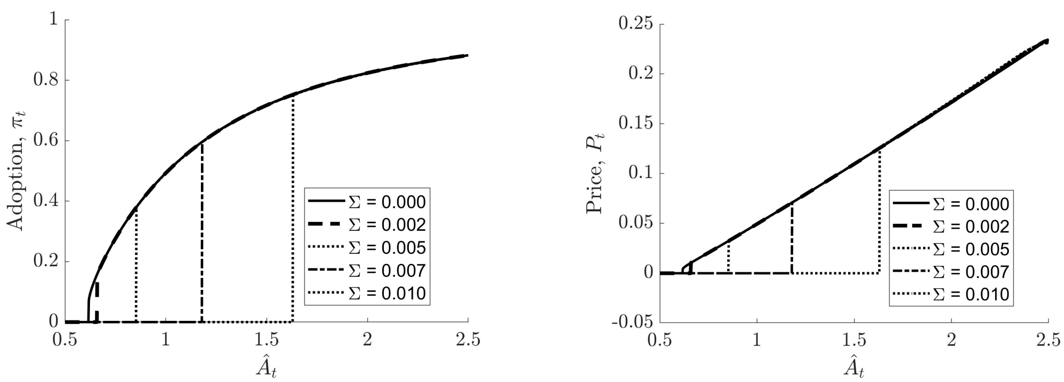

Figure 3: Fraction of agents who adopt, π_t, and token price p_t for different conditional beliefs $\hat{A}_t$ plotted for different values of uncertainty Σ. The parameters for the plot are $r = 0.10,\ \gamma = \frac{1}{r} = 10,\ \sigma_D = 0.5,\ \alpha = 1 \times 10^{-2},\ \log \kappa_i \sim \mathcal{N}(0, 2)$.

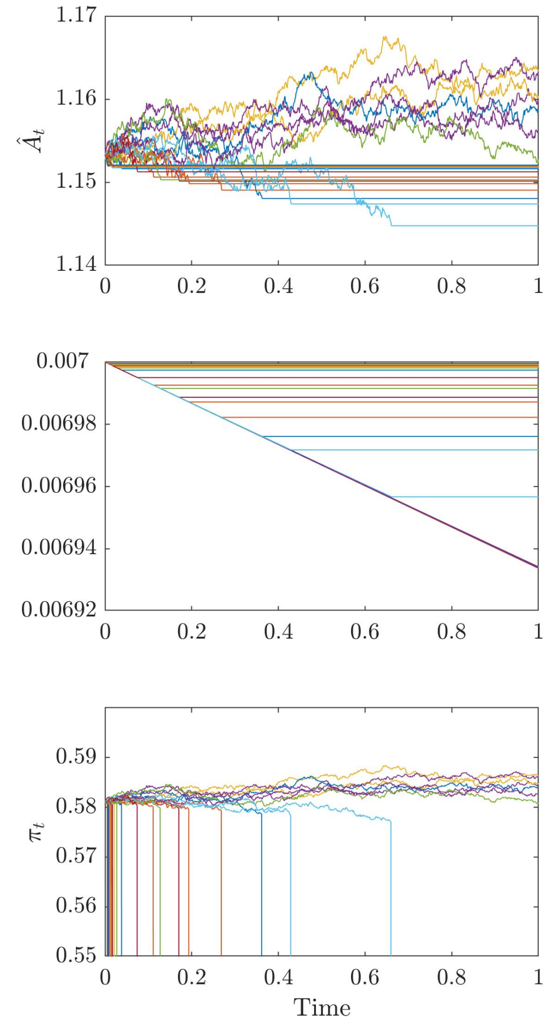

Figure 4: Simulated paths of the conditional expectation of $A, \hat{A}_t$ (Panel A), the conditional uncertainty about A, Σ_t (Panel B), and the fraction of users that adopt, π_t, (Panel C) over time. The parameters for the plot are $r = 0.10$, $\gamma = \frac{1}{r} = 10$, $\sigma_D = 0.5$, $\alpha = 1 \times 10^{-2}$, $\log \kappa_i \sim \mathcal{N}(0, 2)$.

Appendix

Proof of Proposition 1:

Step 1: The Convex Dual Problem for User i

We first rewrite user i's law of motion of wealth dW_{it} from equation 7 based on public information (i.e., under $\mathcal{F}_t^c$) as

$$\begin{aligned} dW_{it} &= \Big(rW_{it} + \omega_{it}W_{it}\Big[\left(a_{it} + \alpha\left(1 - a_{it}\right)\right)\pi_t \hat{A}_t/P_t + \mu_{Pt} - r\Big] - \pi_t \kappa_i a_{it}\Big)\,dt \\ &\quad - c_{it}dt + \omega_{it}W_{it}\left(\left(a_{it} + \alpha\left(1 - a_{it}\right)\right)\sigma_D/P_t + \sigma_{Pt}\right)d\hat{Z}_t^A. \end{aligned} \tag{A.1}$$

We can write the optimization problem of user i as the Lagrangian

$$\begin{aligned} U_0^i &= \sup_{c_i,\omega_i,a_i} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(u\left(c_{it}\right)dt - \Lambda_{it}dW_{it} - \Lambda_{it}\left(\pi_t\kappa_i a_{it} + c_{it}\right)dt\right)\right] \qquad \text{(A.2)} \\ &\quad + \mathbb{E}\left[\int_0^\infty e^{-\rho t}\Lambda_{it}\omega_{it}W_{it}\left(\left(\alpha + \left(1-\alpha\right)a_{it}\right)\sigma_D/P_t + \sigma_{Pt}\right)d\hat{Z}_t^A\right] \\ &\quad + \mathbb{E}\left[\int_0^\infty e^{-\rho t}\Lambda_{it}\Big(rW_{it} + \omega_{it}W_{it}\Big(\left(\alpha + \left(1-\alpha\right)a_{it}\right)\pi_t\hat{A}_t/P_t + \mu_{Pt} - r\Big)\Big)dt\right], \end{aligned}$$

for Lagrange multiplier $\Lambda_{it} \geq 0$. The constraint requires that the wealth of user i (weakly) follows its law of motion. For ease of exposition, we ignore the Lagrangian terms for the laws of motion of $\hat{A}_t$ and Σ_t, as they are immaterial in this characterization.

Let us conjecture that the Lagrange multiplier Λ_{it} (which is the user's stochastic discount factor (SDF)) has the law of motion

$$d\Lambda_{it}/\Lambda_{it} = \mu_{\Lambda t}dt - \sigma_{\Lambda t}d\hat{Z}_t^A. \tag{A.3}$$

This is the relevant costate. Integration by parts yields a saddlepoint problem

$$U_0^i = \sup_{c_i,\omega_i,a_i} \inf_{\Lambda_{it}} \quad \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(u\left(c_{it}\right) - \Lambda_{it}\left(\pi_t \kappa_i a_{it} + c_{it}\right)\right) dt\right] + \Lambda_{i0} W_{i0} \tag{A.4}$$
$$-\mathbb{E}\left[\int_0^\infty e^{-\rho t} \Lambda_{it} W_{it}\left(\omega_{it} \sigma_{\Lambda t}\left(\left(\alpha + (1-\alpha) a_{it}\right) \sigma_D / P_t + \sigma_{Pt}\right)\right) dt\right]$$
$$+\mathbb{E}\left[\int_0^\infty e^{-\rho t} \Lambda_{it} \omega_{it} W_{it}\left(\left(\alpha + (1-\alpha) a_{it}\right) \pi_t \hat{A}_t / P_t + \mu_{Pt} - r\right) dt\right]$$
$$+\mathbb{E}\left[\int_0^\infty e^{-\rho t} \Lambda_{it} W_{it}\left(\mu_{\Lambda t} + r - \rho\right) dt\right] - \lim_{T \to \infty} \mathbb{E}\left[e^{-\rho T} \Lambda_{iT} W_{iT}\right].$$

Applying complementary slackness to the problem (A.4) with respect to wealth W_{it}, terminal wealth W_{iT}, and user investment ω_{it} impose

$$\mu_\Lambda = -\left(r - \rho\right), \tag{A.5}$$
$$\sigma_\Lambda\left(a_i\right) = \frac{\left(\alpha + (1-\alpha) a_i\right) \pi \hat{A} + \mu_P P - rP}{\left(\alpha + (1-\alpha) a_i\right) \sigma_D + \sigma_P P}, \tag{A.6}$$

and the transversality condition

$$\lim_{T \to \infty} \mathbb{E}\left[e^{-\rho T} \Lambda_{iT} W_{iT}\right] = 0. \tag{A.7}$$

The optimal program, given (A.5), (A.6), and (A.7), reduces to

$$U_0^i = \sup_{c_i,a_i} \inf_{\Lambda_{it}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(u\left(c_{it}\right) - \Lambda_{it}\left(\pi_t \kappa_i a_{it} + c_{it}\right)\right) dt\right] + \Lambda_{i0} W_{i0}. \tag{A.8}$$

We next assume that the Minimax Theorem holds and that we can interchange the $\sup_{c_i,\omega_i,a_i} \inf_{\Lambda_{it}}$ with $\inf_{\Lambda_{it}} \sup_{c_i,\omega_i,a_i}$. We can then first maximize over the user's consumption to find that

$$c_{it} = u'^{-1}\left(\Lambda_t^i\right) = \frac{1}{\gamma} \log \gamma - \frac{1}{\gamma} \log \Lambda_{it}, \tag{A.9}$$

for CARA utility. We will solve for the optimal adoption policy a_i after solving for optimally adjusted wealth. This is because a change in the adoption policy also changes the state price deflator, Λ_{it}, of optimally invested wealth.

Step 2: Optimally Invested Wealth and Investment Policy

Let us conjecture that optimally invested wealth, F_{it}, that finances consumption and adoption costs

$$F_{i0} = \mathbb{E}\left[\int_0^\infty e^{-\rho t}\frac{\Lambda_{it}}{\Lambda_0}\left(c_{it} + \pi_t\kappa_i a_{it}\right)dt\right]. \tag{A.10}$$

is a Markov process in the three state variables $\left(\Lambda_{it}, \hat{A}_t, \Sigma_t\right)$ or $F_{it} = F_i\left(\Lambda_{it}, \hat{A}_{it}, \Sigma_t\right)$. Notice that

$$\tilde{F}_{it} = \mathbb{E}\left[\int_0^t e^{-\rho s}\Lambda_{is}\left(c_{is} + \pi_s\kappa_i a_{is}\right)ds + e^{-\rho t}\Lambda_{it}F_{it}\right] \tag{A.11}$$

is a deflated gains process, and therefore a martingale

$$\begin{aligned} d\tilde{F}_{it} &= e^{-\rho t}\Lambda_{it}\left(c_{it} + \pi_t\kappa_i a_{it}\right)dt + d\left(e^{-\rho t}\Lambda_{it}F_{it}\right) \\ &= e^{-\rho t}\Lambda_{it}\left(c_{it} + \pi_t\kappa_i a_{it}\right)dt + d\left(e^{-\rho t}\Lambda_{it}\right)F_{it} + e^{-\rho t}\Lambda_{it}dF_{it} + d\left\langle e^{-\rho t}\Lambda_{it}, F_{it}\right\rangle, \end{aligned} \tag{A.12}$$

and taking expectations with $\mu_{\Lambda_i} = -\left(r - \rho\right)$ and suppressing t subscripts

$$c_i + \pi\kappa_i a_i + \frac{1}{dt}\mathbb{E}\left[dF_i\right] = rF_i + \sigma_{\Lambda_i}\sigma_{F_i}, \tag{A.13}$$

where σ_{F_i} is the diffusion of F_i. Because F_i is Markov in $\left(\Lambda_{it}, \hat{A}_t, \Sigma_t\right)$, by Itô's Lemma, F_i also has law of motion

$$dF_i = \left(\partial_{\Lambda_i} F_i \Lambda_i \mu_{\Lambda_i} + \frac{1}{2}\partial_{\Lambda_i\Lambda_i} F_i \Lambda_i^2 \sigma_{\Lambda_i}^2 - \partial_{\Sigma} F_i \left(\frac{\pi\Sigma}{\sigma_D}\right)^2 + \frac{1}{2}\partial_{\hat{A}\hat{A}} F_i \left(\frac{\pi\Sigma}{\sigma_D}\right)^2\right) dt$$
$$+ \left(-\Lambda_i \partial_{\Lambda_i} F_i \frac{(\alpha + (1-\alpha) a_i) \pi \hat{A}/P + \mu_P}{(\alpha + (1-\alpha) a_i) \sigma_D/P + \sigma_P} + \partial_{\hat{A}} F_i \frac{\pi\Sigma}{\sigma_D}\right) d\hat{Z}^A, \quad \text{(A.14)}$$

so that

$$\sigma_{F_i} = -\Lambda_i \partial_{\Lambda_i} F_i \sigma_{\Lambda_i} + \partial_{\hat{A}} F_i \frac{\pi\Sigma}{\sigma_D}. \quad \text{(A.15)}$$

Matching diffusion terms with the laws of motion of wealth (A.1) and optimal invested wealth (A.14) to arrive at

$$\omega_{it} F_i \left((\alpha + (1-\alpha) a_i) \sigma_D/P + \sigma_P\right) d\hat{Z}^A = \left(-\Lambda_i \partial_{\Lambda_i} F_i \sigma_{\Lambda_i} + \partial_{\hat{A}} F_i \frac{\pi\Sigma}{\sigma_D}\right) d\hat{Z}^A, \quad \text{(A.16)}$$

from which follows given σ_{Λ_i} from (A.6) that

$$\omega_i = -\Lambda_i \frac{\partial_{\Lambda_i} F_i}{F_i} \frac{(\alpha + (1-\alpha) a_i) \pi \hat{A} + \mu^P P - rP}{\left((\alpha + (1-\alpha) a_i) \sigma_D + \sigma_P P\right)^2} P + \frac{\partial_{\hat{A}} F_i}{F_i} \frac{\frac{\pi\Sigma}{\sigma_D}}{(\alpha + (1-\alpha) a_i) \sigma_D + \sigma_P P} P. \quad \text{(A.17)}$$

Matching the drift terms of (A.13), (A.14), (A.15), and substituting with (A.9) and $\mu_{\Lambda_i} = -(r-\rho)$, we arrive at the dual HJB variational inequality

$$0 = \frac{1}{\gamma} \log \gamma - \frac{1}{\gamma} \log \Lambda_i + \pi \kappa_i a_i - \partial_{\Lambda_i} F_i \Lambda_i (r - \rho) - \partial_{\Sigma} F_i \left(\frac{\pi\Sigma}{\sigma_D}\right)^2 + \frac{1}{2}\partial_{\Lambda_i\Lambda_i} F_i \Lambda_i^2 \sigma_{\Lambda_i}^2$$
$$+ \frac{1}{2}\partial_{\hat{A}\hat{A}} F_i \left(\frac{\pi\Sigma}{\sigma_D}\right)^2 - \sigma_{\Lambda_i} \left(-\Lambda_i \partial_{\Lambda_i} F_i \sigma_{\Lambda_i} + \partial_{\hat{A}} F_i \frac{\pi\Sigma}{\sigma_D}\right) - rF_i. \quad \text{(A.18)}$$

Let us conjecture that

$$F_i = f_0 + f_{\Lambda_i} \log \Lambda_i + f_i (A, \Sigma). \quad \text{(A.19)}$$

Because prices do not depend on the Λ_i of any one user, it follows by substituting equation (A.19) into equation (A.18) that

$$f_{\Lambda_i} = -\frac{1}{r\gamma}, \tag{A.20}$$

$$f_0 = \frac{1}{r\gamma}\left(\log\gamma + \frac{r-\rho}{r}\right). \tag{A.21}$$

and that $f_i(A,\Sigma)$ satisfies

$$0 = \inf_{a_i}\left\{\pi\kappa_i a_i - \frac{1}{2}\frac{1}{r\gamma}\sigma_{\Lambda_i}^2 - \partial_{\hat{A}} f_i \sigma_{\Lambda_i}\frac{\pi\Sigma}{\sigma_D}\right\} - \partial_\Sigma f_i\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 + \frac{1}{2}\partial_{\hat{A}\hat{A}} f_i\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 - r f_i. \tag{A.22}$$

Since this relation holds for optimally invested wealth, f_i is strictly a sub-martingale under any suboptimal adoption policy because the cost being financed is increasing over time. We recognize the decision to adopt is reversible at any instant, and consequently the derivatives of f_i do not instantaneously "jump" with changes in the adoption decision.

Let $x_i = \frac{\omega_i F_i}{P}$ be the number tokens that user i' purchases, then substituting our conjecture for F_i into (A.17), we arrive at

$$x_i = \frac{1}{r\gamma}\frac{(\alpha+(1-\alpha)a_i)\pi\hat{A} + \mu_P P - rP}{((\alpha+(1-\alpha)a_i)\sigma_D + \sigma_P P)^2} + \partial_{\hat{A}} f_i \frac{\frac{\pi\Sigma}{\sigma_D}}{(\alpha+(1-\alpha)a_i)\sigma_D + \sigma_P P}. \tag{A.23}$$

Let us distinguish x_i^a and x_i^s to be the demand of adopters and speculators

$$x_i^a = \frac{1}{r\gamma}\frac{\pi\hat{A} + \mu^P P - rP}{(\sigma_D + \sigma_P P)^2} + \partial_{\hat{A}} f_i \frac{\frac{\pi\Sigma}{\sigma_D}}{\sigma_D + \sigma_P P}, \tag{A.24}$$

$$x_i^s = \frac{1}{r\gamma}\frac{\alpha\pi\hat{A} + \mu^P P - rP}{(\alpha\sigma_D + \sigma_P P)^2} + \partial_{\hat{A}} f_i \frac{\frac{\pi\Sigma}{\sigma_D}}{\alpha\sigma_D + \sigma_P P}. \tag{A.25}$$

Since adopters earn a convenience yield and an expected excess payoff, then they have a higher demand than speculators for token. As such, $x_i^a \geq 0$.

Step 3: Optimal Adoption Policy

The optimal policy from (A.22) then reduces to adopt if $\kappa_i \leq \kappa^*$ where

$$\begin{aligned} \kappa^* &= \frac{\sigma_{\Lambda_i}(1)^2 - \sigma_{\Lambda_i}(0)^2}{2\pi r\gamma} + \partial_{\hat{A}} f_i \frac{\Sigma}{\sigma_D} \left(\sigma_{\Lambda_i}(1) - \sigma_{\Lambda_i}(0)\right) \\ &= \frac{SR(1)^2 - SR(0)^2}{2\pi r\gamma} + \partial_{\hat{A}} f_i \frac{\Sigma}{\sigma_D} \left(SR(1) - SR(0)\right), \end{aligned} \tag{A.26}$$

because σ_{Λ_i} is the Sharpe Ratio $SR(a_i)$. A subtlety is that, for the marginal adopter, $\partial_{\hat{A}} f_i$ is the same whether the marginal user adopts or speculates by continuity and smooth-pasting because adopting represents exercising a (reversible) option. For adopters for whom $\kappa_i \neq \kappa^*$, we recognize that

$$\begin{aligned} \partial_{\hat{A}} f_{it} \frac{\pi_t \Sigma_t}{\sigma_D} &= \underbrace{-\frac{1}{2r\gamma}\mathbb{E}\left[\int_t^\infty e^{-r(s-t)} \partial_{\hat{A}} \left(SR_{is}\right)^2 \mathcal{D}_t \hat{A}_s ds\right]}_{\textit{Expected Impulse to Future Squared Sharpe Ratios}} \\ &\quad \underbrace{+\kappa_i \mathbb{E}\left[\int_t^\infty e^{-r(s-t)} \partial_{\hat{A}} \left(\pi_t \mathbf{1}_{\{\kappa_i = \kappa_s^*\}}\right) \mathcal{D}_t \hat{A}_s ds\right]}_{\textit{Expected Impulse to Future Adoption Costs}}, \end{aligned} \tag{A.27}$$

where $\mathcal{D}_t \hat{A}_s > 0$. These two terms are increasing in κ_i for adopters because the higher the participation cost, the more likely the user is to be a speculator. As a result, higher κ_i adopters have a lower expected discounted future adoption costs if $\hat{A}$ falls as well as less sensitivity to the decrease in the adopter Sharpe Ratio. Intuitively, the wealth of adopters closest to the κ^* threshold is most sensitive to changes in the perceived platform fundamental. Rewriting the adoption decision of a user with participation cost κ_i who adopts as

$$\kappa_i < \kappa^* + \left(\partial_{\hat{A}} f_i - \partial_{\hat{A}} f_*\right) \frac{\Sigma}{\sigma_D} \left(SR\left(1\right) - SR\left(0\right)\right) < \kappa^*, \tag{A.28}$$

It follows that a user adopts if $\kappa_i \leq \kappa^*$.

Notice that by completing the square

$$\frac{SR\left(a_i\right)^2}{2\pi r\gamma} + \partial_{\hat{A}} f_i \frac{\Sigma}{\sigma_D} SR\left(a_i\right) = \frac{\left(SR\left(1\right) + r\gamma\partial_{\hat{A}} f_i \frac{\pi\Sigma}{\sigma_D}\right)^2}{2\pi r\gamma} - \frac{1}{2\pi r\gamma}\left(r\gamma\partial_{\hat{A}} f_i \frac{\pi\Sigma}{\sigma_D}\right)^2, \tag{A.29}$$

from which follows that the optimal adoption decision with speculators from equation (A.26) satisfies

$$\kappa^* = \frac{\left(SR\left(1\right) + r\gamma\partial_{\hat{A}} f_* \frac{\pi\Sigma}{\sigma_D}\right)^2}{2\pi r\gamma} - \frac{\left(SR\left(0\right) + r\gamma\partial_{\hat{A}} f_* \frac{\pi\Sigma}{\sigma_D}\right)^2}{2\pi r\gamma}, \tag{A.30}$$

since $r\gamma\partial_{\hat{A}} f_* \frac{\pi\Sigma}{\sigma_D}$ is the same whether the user adopts or speculates because of smooth-pasting. Notice that no adopter would pay the fixed adoption flow cost and hold tokens to just speculate in the token.

For speculators, $\partial_{\hat{A}} f_i$ is negative if speculators are long tokens and positive if speculators are short tokens. This derivative is also increasing in κ_i for speculators because the higher the κ_i, the lower the likelihood that the first term switches to that of an adopter, in which case we know that the Sharpe Ratio is higher with adoption, otherwise no adopter would adopt. The second term is also increasing in κ_i for some range of $\kappa_i > \kappa^*$, the flow adoption cost must be increasing in κ_i. For κ_i beyond this range, the κ_i must be sufficiently high that user i would never adopt, otherwise the κ_j just below would adopt and then we can repeat the previous argument. As such, speculators for whom

$\kappa_i > \kappa^*$ choose to speculate.

Because this is independent of Λ_i it confirms our conjecture about the functional form of F_i. a_i can then be expressed as

$$a_i = \mathbf{1}_{\{\kappa_i \leq \kappa^*\}}. \tag{A.31}$$

We relegate establishing the transversality condition holds and sufficiency to the Online Appendix.

Proof of Proposition 2:

Substituting with x_i^a and x_i^s from Proposition 1 and equations (17) and (18) into equation (16), we arrive at the equation (19).

Notice now that if speculators take a positive position in tokens, then equation (A.30) implies it is necessary for an equilibrium to exist that

$$SR\left(1\right) \geq SR\left(0\right). \tag{A.32}$$

Consequently, this is the necessary condition for existence of an equilibrium at time t. Notice that when there is uncertainty ($\Sigma_t > 0$), this can be rewritten

$$\frac{\pi \hat{A}}{\sigma_D} \geq \frac{\mu^P P - rP}{\sigma_P P}, \tag{A.33}$$

when $P > 0$ because $\frac{y\pi\hat{A} + \mu_P P - rP}{y\sigma_D + \sigma_P P}$ is larger when $y = 1$ than 0 if $P > 0$.

In contrast, if speculators take a negative position in the token, then $\partial_{\hat{A}} f_* = 0$ for the critical κ^* because the hedging demand of the user switches sign depending on whether it is an adopter (and long the asset) or a speculator (and short the asset). At the switching point then, by continuity of the first derivative, it must be the case that $\partial_{\hat{A}} f_* = 0$. In this situation, the necessary condition for the existence of an equilibrium is now

$$SR\left(1\right) \geq -SR\left(0\right), \tag{A.34}$$

because $SR(1) \geq 0$ and shorting implies a negative Sharpe Ratio. This implies a condition (because $\mu^P P - rP < 0$ for speculators to short) of

$$\frac{\pi \hat{A}}{\sigma_D} \geq \frac{1 + \alpha + 2\sigma_P P/\sigma_D}{1 + \alpha + 2\alpha\sigma_D/\sigma_P P} \left| \frac{\mu^P P - rP}{\sigma_P P} \right|. \tag{A.35}$$

As $\alpha \to 0$, this condition becomes

$$\frac{\pi \hat{A}}{\sigma_D} \geq \left(1 + \frac{2\sigma_P P}{\sigma_D}\right) \left| \frac{\mu^P P - rP}{\sigma_P P} \right| > \left| \frac{\mu^P P - rP}{\sigma_P P} \right|. \tag{A.36}$$

because $\sigma_P P \geq 0$. Thus, for α sufficiently small, this condition is necessary. If this condition is not satisfied, then $\pi_t = 0$ and price collapses to a constant

$$P = -\gamma (\alpha \sigma_D)^2 . \tag{A.37}$$

Proof of Proposition 3:

In the absence of participation fees, all users adopt the platform's services. In this case, optimally invested risk-adjusted wealth is independent of fluctuations in A, and the token price is

$$P = \frac{1}{r}\hat{A} + p(\Sigma) - \gamma\sigma_D^2, \tag{A.38}$$

where $p(\Sigma)$ satisfies

$$r\sigma_D^2 \frac{1}{\Sigma^2} p(\Sigma) + p'(\Sigma) = -2\gamma\sigma_D^2 \frac{1}{\Sigma} - \frac{\gamma}{r}. \tag{A.39}$$

By the method of integrating factors, we can rewrite equation (A.39) as

$$\left[\exp\left(-r\sigma_D^2 \frac{1}{\Sigma}\right) p(\Sigma)\right]' = -\gamma \left(2\sigma_D^2 \frac{1}{\Sigma} + \frac{1}{r}\right) \exp\left(-r\sigma_D^2 \frac{1}{\Sigma}\right). \tag{A.40}$$

Defining $y = -\frac{r\sigma_D^2}{\Sigma}$, then equation (A.40) has the solution because $p(0) = 0$ of

$$p(\Sigma) = \gamma\sigma_D^2 \exp\left(\frac{r\sigma_D^2}{\Sigma}\right) \int_{-\infty}^{-\frac{r\sigma_D^2}{\Sigma}} \left(\frac{2}{y'} - \frac{1}{y'^2}\right) \exp(y')\, dy'. \tag{A.41}$$

In this situation, $\pi_t = 1$ for all t and $\Sigma_t/\sigma_A^2 = \left(1 + \left(\frac{\sigma_A}{\sigma_D}\right)^2 t\right)^{-1}$.

Proof of Proposition 4:

When there is no uncertainty and $\Sigma = 0$, then prices are constant (i.e., $\mu_P P = \sigma_P P = 0$) and there is no hedging demand (i.e., $\partial_{\hat{A}} f_i \equiv 0$). It is then straightforward to see that the token price from Proposition 2 collapses to

$$P = \frac{\alpha}{r}\frac{1-(1-\alpha)\,\pi}{1-(1-\alpha^2)\,\pi}\pi A - \frac{\alpha^2\gamma\sigma_D^2}{1-(1-\alpha^2)\,\pi}, \tag{A.42}$$

It is straightforward to see that the coefficient on A, $\frac{\alpha}{r}\frac{1-(1-\alpha)\pi}{1-(1-\alpha^2)\pi}\pi$, is increasing in π, while the second term, $-\frac{\gamma\alpha^2\sigma_D^2}{1-(1-\alpha^2)\pi}$, is decreasing in π.

From equation (14), with some manipulation one can see that the optimal adoption threshold is now given by

$$\pi\kappa^* = \frac{1}{2r\gamma\sigma_D^2}\frac{1-\alpha}{\alpha}rP\left((1-\alpha)\,\frac{1-(1+\alpha)\,\pi}{1-(1-\alpha^2)\,\pi}\pi A + \frac{(1+\alpha)\,\alpha r\gamma\sigma_D^2}{1-(1-\alpha^2)\,\pi}\right), \tag{A.43}$$

as all uncertainty about A has resolved. It is clear that users again adopt a cutoff strategy in deciding whether to adopt.

In what follows, suppose $\pi \le (1+\alpha)^{-1}$. It is then clear for $\kappa^* \ge 0$ that $P \ge 0$ if an equilibrium exists. Define $y = r\gamma\sigma_D^2$ and $\tilde{A} = A/y$. Given $\pi = G(\kappa^*)$, it follows substituting for equation (A.43) that a solution solves:

$$(1-\alpha)\,\frac{(1-\alpha)\left(1-2\pi+\left(1-\alpha^2\right)\pi^2\right)\left(\pi\tilde{A}\right)^2 + 2\alpha^2\pi\tilde{A} - \alpha^2\,(1+\alpha)}{\left(1-\left(1-\alpha^2\right)\pi\right)^2} = \pi G^{-1}(\pi)\,. \tag{A.44}$$

The right-hand side of equation (A.44) is strictly increasing in π from 0 to $\frac{1}{1+\alpha}G^{-1}\left(\frac{1}{1+\alpha}\right)$ as π increases from 0 to $\frac{1}{1+\alpha}$, where $\frac{1}{1+\alpha}G^{-1}\left(\frac{1}{1+\alpha}\right)$ is arbitrarily large because G has a support on $[0,\infty)$. This is because by the

Implicit Function Theorem, $\frac{d}{d\pi}G^{-1}(\pi) = G\left(G^{-1}(\pi)\right)^{-1} > 0$. Let the left-hand side of equation (A.44) be H. It has a bound of $-\alpha^2(1-\alpha^2)$ for $\pi = 0$ and $(1-\alpha^2)\left(\frac{2\tilde{A}}{(1+\alpha)^2} - 1\right) < \infty$ for $\pi = \frac{1}{1+\alpha}$. Consequently, the two curves must intersect an even number of times.

We next establish there are only two equilibria when equilibria exist. At the first equilibrium, $H > 0$ for equation (A.44) to be satisfied and $\partial_\pi H > 0$. At the second equilibrium, $H > 0$ but now $\partial_\pi H < 0$. Notice now that:

$$\frac{1-(1-\alpha^2)\pi}{2(1-\alpha)}\partial_\pi H = (1+\alpha)H + \frac{(1-\alpha)\left(1-3\pi+2\left(1-\alpha^2\right)\pi^2\right)\pi\tilde{A}^2 + \alpha^2\tilde{A}}{1-(1-\alpha^2)\pi}. \tag{A.45}$$

The second term of equation (A.45) determines the shape of the distribution H. Notice when there is an equilibrium, then there are at least two equilibria. This implies that the second term has to be downward-sloping in π on $\left[0, \frac{1}{1+\alpha}\right]$ (at least initially) from positive to negative. The second term, in this case, is either downward-sloping or U-shaped. If it is downward-sloping, then $\partial_\pi H$ cannot be become positive again for a third equilibrium. If it is U-shaped and becomes positive again, then $\partial_\pi H$ could become positive for a third equilibrium, but it would not be possible for the second term then to become more negative for $\partial_\pi H < 0$ for a fourth equilibrium. Thus, there are either zero or two equilibria for each A. Our focus is on the higher π equilibrium.

Next, we recognize that we can rewrite equation (A.44) in equilibrium as

$$B = \pi G^{-1}(\pi) - H \equiv 0. \tag{A.46}$$

Applying the Implicit Function Theorem to equation A.46, then

$$\frac{d\pi}{d\tilde{A}} = \frac{\partial_{\tilde{A}} H}{G^{-1}(\pi) + \pi / G\left(G^{-1}(\pi)\right) - \partial_\pi H}. \tag{A.47}$$

Notice from equation A.44 for $\pi \leq \frac{1}{1+\alpha}$,

$$\partial_{\tilde{A}} H = \frac{2\pi(1-\alpha)}{\left(1-(1-\alpha)^2\pi\right)^2}\left(\alpha^2 + (1-\alpha)\pi\left((1-\pi)^2 - \alpha^2\pi^2\right)\tilde{A}\right) > 0. \tag{A.48}$$

Notice, when an equilibrium exists, it must be the case that $\partial_\pi H > 0$ for odd equilibria and $\partial_\pi H < 0$ for even equilibria by the above arguments. In even equilibria, we consequently have that $\partial_{\tilde{A}}\pi > 0$, and consequently $\partial_A \pi > 0$. As $\partial_A \pi > 0$ is the natural comparative static (i.e., a better platform has more participants), it follows even equilibria are stable, and consequently odd equilibria (as the alternating equilibria) are unstable.

Finally, notice if there is not a high participation (high π) equilibrium, then there are zero equilibria. In the high participation equilibrium, π is increasing in A. Since the equilibrium is continuous, it follows when A is low, then π is low and network effects become fragile. From equation (A.45) then, when $\tilde{A}$ is sufficiently low that $\partial_\pi H < 0$ at $\pi = 0$, it is impossible for there to be an equilibrium (which requires $\partial_\pi H > 0$). Consequently, there exists a cutoff A^c such that if $A \leq A^c$, then no equilibrium on the platform exists.

Proof of Proposition 5:

In the absence of speculators, when $\Sigma_t = 0$, the critical type κ^* is given by

$$\kappa^* = \frac{1}{2\pi r\gamma}\left(\frac{\pi A - rP}{\sigma_D}\right)^2 = \frac{r\gamma}{2\pi}\left(\frac{\sigma_D}{\pi}\right)^2. \tag{A.49}$$

From Proposition 4, π is (weakly) increasing in A with speculators and it is a constant without. As such, for A less than some A_*, adoption is lower with speculators while if A (weakly) exceeds A_*, then adoption is higher. In addition, π and πA are ex ante more volatile with speculators because π is a constant without speculators and positively comoves with A with speculators.

Consider the case when $\Sigma_t > 0$. In this case, those that do not adopt hold only the risk-free asset. This implies that their optimally invested wealth has no diffusion process, or by Itô's Lemma

$$dF_{it} - \mathbb{E}\left[dF_{it} \mid \mathcal{F}_t\right] = \left(\partial_{\hat{A}} F_{it} \frac{\pi_t \Sigma_t}{\sigma_D} - \partial_{\Lambda} F_{it} \Lambda_{it} \sigma_{\Lambda_i t}\right) d\hat{Z}_t^A \equiv 0, \tag{A.50}$$

in the no adoption region of $\left(\hat{A}, \Sigma\right)$. Given F_t from equation (A.19), $\sigma_{\Lambda_i} = -r\gamma \partial_{\hat{A}} f_i \frac{\pi\Sigma}{\sigma_D}$ and the dual HJB equation in the no adoption region is

$$0 = \frac{1}{2}\frac{1}{r\gamma}\sigma_{\Lambda_i}^2 - \partial_{\Sigma} f_i \left(\frac{\pi\Sigma}{\sigma_D}\right)^2 + \frac{1}{2}\partial_{\hat{A}\hat{A}} f_i \left(\frac{\pi\Sigma}{\sigma_D}\right)^2 - r f_i. \tag{A.51}$$

A non-adopting user's wealth is dragged down by the volatility of her SDF and is not compensated for by a risk premium from holding tokens. This is because she may invest in tokens in the future, in which case there is uncertainty in her consumption. This volatility differs across non-adopting users because markets are effectively incomplete for them since they cannot trade $\hat{A}_t$ risk.

Defining $SR_i(0) = \sigma_{\Lambda_i} = -r\gamma \partial_{\hat{A}} f_* \frac{\pi\Sigma}{\sigma_D}$ to be the Sharpe Ratio for the marginal user if she does not adopt, then the threshold from Proposition 1 is

$$\kappa^* = \frac{1}{\pi}\left(\frac{SR(1)^2}{2r\gamma} + \partial_{\hat{A}} f_* \frac{\pi\Sigma}{\sigma_D} SR(1) + \frac{1}{2r\gamma} SR(0)^2\right) = \frac{\left(SR(1) + r\gamma \partial_{\hat{A}} f_* \frac{\pi\Sigma}{\sigma_D}\right)^2}{2\pi r\gamma} > 0. \tag{A.52}$$

Since $\kappa^* > 0$, $\pi > 0$ and learning occurs on the platform in finite time.

Online Appendix for A Theory of Speculation in Community Assets

Kevin Mei and Michael Sockin

Online Appendix A: Additional Proofs

Continuation of Proof of Proposition 1:

Step 4: Transversality

The transversality condition (A.7) then becomes

$$\lim_{T\to\infty} \mathbb{E}\left[e^{-\rho T}\Lambda_{iT}F_{iT}\right] = 0. \tag{IA.1}$$

Notice that the Sharpe Ratio $SR\left(a_i\right)$ is finite $a.s.$ because A is finite $a.s.$ and the Sharpe Ratio is 0 if $\sigma_P P = 0$. As such, $e^{-\rho T}\Lambda_{iT} = e^{-rT}\tilde{\Lambda}_{iT}$, where $\tilde{\Lambda}_{iT}$ has expectation Λ_{i0} and is finite $a.s.$ Substituting for F_{iT}, the transversality condition (IA.1) becomes

$$\begin{aligned}\lim_{T\to\infty} \mathbb{E}\left[e^{-\rho T}\Lambda_{iT}F_{iT}\right] &= \lim_{T\to\infty} \frac{1}{r\gamma}\left(\log\gamma + \frac{r-\rho}{r}\right)e^{-rT} - \frac{\rho - r}{r\gamma}\Lambda_{i0}e^{-rT}T \\ &\quad -\frac{1}{r\gamma}\mathbb{E}\left[e^{-rT}\tilde{\Lambda}_{iT}\log\tilde{\Lambda}_{iT}\right] + \mathbb{E}\left[e^{-\rho T}\Lambda_{iT}f_i\left(\hat{A}_T, \Sigma_T\right)\right] \\ &= \lim_{T\to\infty} \mathbb{E}\left[e^{-\rho T}\Lambda_{iT}f_i\left(\hat{A}_T, \Sigma_T\right)\right],\end{aligned} \tag{IA.2}$$

As such, the transversality condition (IA.1) reduces to

$$\lim_{T\to\infty} \mathbb{E}\left[e^{-\rho T}\Lambda_{iT}f_i\left(\hat{A}_T, \Sigma_T\right)\right] = 0. \tag{IA.3}$$

Step 5: Sufficiency

As is standard, a solution to (13) that satisfies the transversality condition (IA.3) and the appropriate boundary conditions is a solution to the continuation convex dual portfolio choice problem

$$J_0^i = \inf_{\Lambda_{it}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(u\left(c_t\right) - \Lambda_{it}\left(c_i\left(\Lambda_{it}\right) + \pi_t \kappa_i \mathbf{1}_{\{\kappa_i \leq \kappa_t^*\}}\right)\right) dt\right], \qquad \text{(IA.4)}$$

and Λ_{i0} is chosen such that[1]

$$V_0^i = \inf_{\Lambda_{i0}} J_0^i + \Lambda_{i0} W_{i0}. \qquad \text{(IA.5)}$$

It is also standard that under standard regularity conditions, the solution to the convex dual problem is a solution to the saddle-point problem and a solution to the primal problem (8) according to

$$U_0^i = V_0^i + \Lambda_{i0} F_{i0}. \qquad \text{(IA.6)}$$

As adoption is effectively a stopping time problem, optimality of the adoption region imposes both continuity (value-matching) and smoothness (smooth-pasting) at the boundary $\hat{A}^*(\Sigma)$ (equivalently $\Sigma^*\left(\hat{A}\right)$) at which the user transitions from speculator to adopter. Otherwise, the user is not indifferent between policies at the boundary. This imposes

$$\begin{aligned} \lim_{\hat{A} \to \hat{A}^*(\Sigma)^-} \partial_{\hat{A}}^j f_i\left(\hat{A}, \Sigma\right) &= \lim_{\hat{A} \to \hat{A}^*(\Sigma)^+} \partial_{\hat{A}}^j f_i\left(\hat{A}, \Sigma\right), \ j \in \{0,1\} \\ \lim_{\Sigma \to \Sigma^*\left(\hat{A}\right)^-} \partial_{\Sigma}^j f_i\left(\hat{A}, \Sigma\right) &= \lim_{\Sigma \to \Sigma^*\left(\hat{A}\right)^+} \partial_{\Sigma}^j f_i\left(\hat{A}, \Sigma\right), \ j \in \{0,1\} \end{aligned} \qquad \text{(IA.7)}$$

which pin down the boundary and the value of $f_i\left(\hat{A}, \Sigma\right)$ at the speculator-adopter transition boundary.

An additional important boundary condition is when $\Sigma = 0$. In this case,

[1] Because there are no wealth effects with CARA utility, this initial optimization is not material for our characterization.

aggregate uncertainty has dissipated and the token price converges to a constant and

$$f_i\left(A,0\right)=\frac{1}{r}\left(\kappa_i-\frac{1}{2r\gamma}\left(\frac{\pi A}{\sigma_D}\right)^2\right)\mathbf{1}_{\{\kappa_i\leq\kappa^*\}}. \tag{IA.8}$$

This is one of the two limiting condition as $T\to\infty$. The other is that the platform collapses at some time $t<T$ and $f_i\left(A,\Sigma_t\right)=0$. It is then immediate that the transversality condition (IA.3) is satisfied.

Proof of Corollary 1:

Suppose $\Sigma_{T_\varepsilon}=\varepsilon>0$ but close to 0. By continuity near the steady-state limit when $\Sigma=0$ from Proposition 4, if the perceived platform fundamental, $\hat{A}$, is sufficiently below $\hat{A}^c$, then the platform must also fail at time T_ε.

Similarly, if $\hat{A}$ is sufficiently above $\hat{A}^c$, then there must also be a positive mass of adopters at time T_ε (i.e., $\pi_{T_\varepsilon}>0$). In this case, the platform survives and users eventually learn the true A.

Suppose now that $\hat{A}$ is in a neighborhood of $\hat{A}^c$, Then, there must be a critical $\hat{A}^*_{T_\varepsilon}$ such that there is positive participation ($\pi_{T_\varepsilon}>0$) if $\hat{A}$ is above this threshold and the platform fails below this threshold. We now argue that $\hat{A}^*_{T_\varepsilon}>A^c$. For Σ_{T_ε} sufficiently close to 0, there is a small risk premium embedded in the token's total return to compensate the marginal adopter for the residual fundamental risk. As the convenience yield is πA when the token price is 0 at $\Sigma=0$, it follows the additional risk premia is from an embedded capital gain $\mu_P P-rP$. As existence requires $\frac{\pi\hat{A}}{\sigma_D}\geq\left|\frac{\mu^P P-rP}{\sigma_P P}\right|$ from Proposition 2, it follows that no equilibrium exists locally for $\hat{A}\leq A^c$ (as participation π is also lower). As such, the region of existence is smaller, and $\hat{A}^*_{T_\varepsilon}>A^c$.

We can repeat these local arguments backward in time over infinitesimal ε increments for $\Sigma_{T_{2\varepsilon}}=2\varepsilon$ and $\Sigma_{T_{3\varepsilon}}=3\varepsilon$ and so on to establish that uncertainty about A narrows the continuation region for $\hat{A}$ for which there is a positive mass of adopters ($\pi_t>0$). Consequently, there exists an adapted process $\hat{A}^c\left(\Sigma_t\right)$ that is increasing in Σ_t such that an equilibrium with positive adoption exists if $\hat{A}_t \geq \hat{A}^c(\Sigma_t)$ and does not exist otherwise.

Further, when $\hat{A}$ (and consequently participation, π) is high, so is the expected convenience yield $\pi\hat{A}$. Since speculators earn a smaller convenience yield than adopters, they take a shorter token position when $\hat{A}$ is high. By continuity, speculators take more positive (less negative) positions when $\hat{A}$ is low and more negative positions when $\hat{A}$ is high.

Proof of Proposition 6:

Step 1: Optimal Issuance Policy for $t > 0$

To abstract from time consistency issues in the platform owner's incentives, we solve for the optimal seigniorage using stochastic variational (Malliavin) calculus tools. Suppose the owner follows an issuance strategy $\{m + \eta\varepsilon_t\}_{t\geq 0}$ for some perturbation ε_t. Then, the law of motion of the platform profits by Itô's Lemma is

$$\begin{aligned} d\Pi &= \left(\partial_M \Pi\,(m+\eta\varepsilon) + \left(\partial_\Sigma \Pi - \frac{1}{2}\partial_{\hat{A}\hat{A}}\Pi\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2\right)dt + \partial_{\hat{A}}\Pi\frac{\pi\Sigma}{\sigma_D}d\hat{Z}^A \\ &= \left(\partial_M \Pi m + \left(\partial_\Sigma \Pi - \frac{1}{2}\partial_{\hat{A}\hat{A}}\Pi\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2\right)dt + \partial_{\hat{A}}\Pi\frac{\pi\Sigma}{\sigma_D}d\tilde{Z}^A, \end{aligned} \tag{IA.9}$$

where

$$d\tilde{Z}^A = d\tilde{Z}^A + \frac{\partial_M \Pi}{\partial_{\hat{A}}\Pi\frac{\pi\Sigma}{\sigma_D}}\eta\varepsilon dt, \tag{IA.10}$$

is a standard Wiener process under a twisted measure $\tilde{\mathcal{P}}$ by applying Girsanov's Theorem to a change of measure $\mathcal{E}_t$

$$\mathcal{E}_t = \exp\left(-\int_0^t \frac{\partial_M \Pi_s}{\partial_{\hat{A}}\Pi_s\frac{\pi\Sigma}{\sigma_D}}\eta\varepsilon_s d\tilde{Z}^A_s - \frac{1}{2}\int_0^t\left(\frac{\partial_M \Pi_s}{\partial_{\hat{A}}\Pi_s\frac{\pi\Sigma}{\sigma_D}}\eta\varepsilon_s\right)^2 ds\right). \tag{IA.11}$$

Under this variation for $t > 0$, platform profit is given by

$$V_0 = P_0 M_0 + \tilde{\mathbb{E}}\left[\int_0^\infty e^{-rt} P_t\left(m_t + \eta\varepsilon_t\right) dt\right] = P_0 M_0 + \mathbb{E}\left[\int_0^\infty e^{-rt} \mathcal{E}_t P_t\left(m_t + \eta\varepsilon_t\right) dt\right], \tag{IA.12}$$

yields the first-order condition at a stationary point (i.e., $\eta = 0$ and $\mathcal{E}_t \equiv 1$)

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\left(P_t m_t\left(-\int_0^t \frac{\partial_M \Pi_s}{\partial_{\hat{A}} \Pi_s \frac{\pi\Sigma}{\sigma_D}} \varepsilon_s d\tilde{Z}_s^A\right) + P_t \varepsilon_t\right) dt\right] = 0, \tag{IA.13}$$

where Π_t is the continuation value

$$\Pi_t = \mathbb{E}\left[\int_t^\infty e^{-r(s-t)} P_s m_s ds\right]. \tag{IA.14}$$

Applying Malliavin Integration-by-Parts to equation (IA.13), we arrive at

$$\mathbb{E}\left[\int_0^\infty \left(\int_0^t \mathcal{D}_s\left(e^{-rt} P_t m_t\right) ds \frac{\partial_M \Pi_t}{\partial_{\hat{A}} \Pi_t \frac{\pi\Sigma}{\sigma_D}} + e^{-rt} P_t\right) \varepsilon_t dt\right] = 0, \tag{IA.15}$$

where $\mathcal{D}$ is the Malliavin derivative operator. We can rewrite equation (IA.15) because of the linearity of $\mathcal{D}$ by rewriting the summation as

$$\begin{aligned} 0 &= \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\frac{\partial_M \Pi_t}{\partial_{\hat{A}} \Pi_t \frac{\pi\Sigma}{\sigma_D}} \mathcal{D}_t\left(\int_t^\infty e^{-r(t-t)} P_s m_s ds\right) + P_t\right) \varepsilon_t dt\right] \\ &= \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\frac{\partial_M \Pi_t}{\partial_{\hat{A}} \Pi_t \frac{\pi\Sigma}{\sigma_D}} \mathcal{D}_t \Pi_t + P_t\right) \varepsilon_t dt\right]. \end{aligned} \tag{IA.16}$$

Since this must hold *a.a.t.*, it follows that the necessary condition for the optimal seigniorage policy from (IA.16) is

$$\frac{\partial_M \Pi_t}{\partial_{\hat{A}} \Pi_t \frac{\pi\Sigma}{\sigma_D}} \mathcal{D}_t \Pi_t + P_t = 0. \tag{IA.17}$$

Given that

$$\mathcal{D}_t \Pi_t = \partial_A \Pi_t \frac{\pi \Sigma}{\sigma_D}, \tag{IA.18}$$

when Π_t is a Markov process by the Clark-Ocone Martingale Representation Theorem, we arrive from (IA.17) at

$$P_t = -\partial_M \Pi_t. \tag{IA.19}$$

Step 2: Optimal Continuation Value for $t > 0$

Notice that the law of motion of Π_t under the optimal issuance policy equation (IA.19) implies that

$$\begin{aligned} d\Pi &= \left(Pm + \partial_M \Pi m + \left(\frac{1}{2} \partial_{\hat{A}\hat{A}} \Pi - \partial_\Sigma \Pi \right) \left(\frac{\pi \Sigma}{\sigma_D} \right)^2 \right) dt + \partial_{\hat{A}} \Pi \frac{\pi \Sigma}{\sigma_D} d\hat{Z}^A \\ &= \left(\frac{1}{2} \partial_{\hat{A}\hat{A}} \Pi - \partial_\Sigma \Pi \right) \left(\frac{\pi \Sigma}{\sigma_D} \right)^2 dt + \partial_{\hat{A}} \Pi \frac{\pi \Sigma}{\sigma_D} d\hat{Z}^A. \end{aligned} \tag{IA.20}$$

Since the deflated gains process for Π_t, $\tilde{\Pi}_t$

$$\tilde{\Pi}_t = \int_0^t P_s m_s ds + e^{-rt} \Pi_t, \tag{IA.21}$$

must be a martingale under the optimal issuance policy, it follows that Π satisfies

$$r\Pi = \left(\frac{1}{2} \partial_{\hat{A}\hat{A}} \Pi - \partial_\Sigma \Pi \right) \left(\frac{\pi \Sigma}{\sigma_D} \right)^2, \tag{IA.22}$$

which along with equation (IA.19) and the appropriate boundary conditions, including transversality

$$\lim_{T \to \infty} \mathbb{E} \left[e^{-rT} \Pi_T \right] = 0. \tag{IA.23}$$

identifies $\Pi = \Pi\left(\hat{A}, \Sigma, M\right)$.

Step 3: Optimal Initial Token Issuance at $t = 0$

Given the definition of $\Pi\left(\hat{A}, \Sigma, M\right)$, the initial token issuance problem at $t = 0$ now solves

$$V_0 = \sup_{M_0} P_0 M_0 + \Pi\left(\bar{A}, \sigma_A^2, M_0\right), \tag{IA.24}$$

it follows that the FOC for the optimal M_0 satisfies

$$M_0 = -\frac{P_0 + \partial_M \Pi\left(\bar{A}, \sigma_A^2, M_0\right)}{\partial_M P_0}. \tag{IA.25}$$

Notice that since $P = -\partial_M \Pi$ for $t > 0$ for the optimal m, equation (IA.25) would be 0 for a continuation discrete issuance $\Delta M_t = M_{t+\Delta t} - M_t$, i.e.,

$$\Delta M_t = -\frac{P_t + \partial_M \Pi_t\left(M_t + \Delta M_t\right)}{\partial_M P_t} = 0. \tag{IA.26}$$

Step 4: User Participation and Token Excess Returns for $t > 0$

Differentiating equation (IA.22) with respect to M, and imposing $P = -\partial_M \Pi$ from equation (IA.19), we find that

$$rP = \left(\frac{1}{2}\partial_{\hat{A}\hat{A}} P - \partial_\Sigma P\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 - \left(\frac{1}{2}\partial_{\hat{A}\hat{A}}\Pi - \partial_\Sigma \Pi\right)\partial_M\left(\frac{\pi\Sigma}{\sigma_D}\right)^2, \tag{IA.27}$$

and imposing $\mu_P P = \left(\frac{1}{2}\partial_{\hat{A}\hat{A}} P - \partial_\Sigma P\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2$ and equation (IA.22), we arrive at

$$\left(\mu_P - r\right)P = 2r\Pi\frac{\partial_M \pi}{\pi}. \tag{IA.28}$$

Consequently, the risk premium on the token price is related to the whether issuing more tokens raises or lowers the volatility of beliefs about A.

This expression also implies that if participation collapses ($\pi = 0$), then there is no risk premium. As the service benefit has zero expectation and is noisy without any participation, it follows no users will hold tokens if $\pi = 0$. Consequently, the platform breaks down if participation collapses.

By similar arguments to those in the proof of Proposition 1, the Sharpe Ratio for users with seigniorage is now

$$SR(a_i) = \frac{(a_i + \alpha(1-a_i))\,\pi\hat{A}/M + (\mu_P - r)\,P}{(a_i + \alpha(1-a_i))\,\sigma_D/M + \sigma_P P} = \frac{(a_i + \alpha(1-a_i))\,\pi\hat{A}/M + 2r\Pi\frac{\partial_M \pi}{\pi}}{(a_i + \alpha(1-a_i))\,\sigma_D/M + \sigma_P P}. \tag{IA.29}$$

By similar arguments to those in the proof of Proposition 2, the necessary condition for positive adoption when α is sufficiently small is now

$$\frac{\pi\hat{A}}{\sigma_D} \geq \left|\frac{(\mu_P - r)\,P}{\sigma_P P}\right| = 2r\left|\Pi\frac{\partial_M \pi/\pi}{\sigma_P P}\right|. \tag{IA.30}$$

Consequently, the owner can control platform participation by affecting contemporaneous participation π and the token's capital gain through the slope of this relation, $\partial_M \pi/\pi$.

Step 5: Optimal Platform Operation

Since the token platform collapses if there is zero participation, in which case the token price is zero (or negative to still attract buyers), the owner will not want to allow participation to collapse. Consequently, the optimal policy is to choose M_0 optimally such that there is positive participation, and then to follow an optimal issuance policy m that maintains it.

Consider the extreme case in which M_0 is arbitrarily large. Then, tokens will have little price appreciation, which suggests that $\frac{\partial_M \pi}{\pi}$ must be approach zero or be negative. This debasement, however, does not detract from the fundamental expected value of the service benefit, $\pi\hat{A}$. As such, even if the token price has minimal price appreciation, there is positive value in collectively adopting it. As such, $\pi > 0$. Consequently, there exist values of M_0 (which may be large) such that there is positive adoption.

Therefore, the platform owner will choose M_0 large enough to foster positive adoption, the exact choice of which is governed by equation (IA.25). At that point, the platform operates with more tokens being issued over time, and is well-defined if M_0 is large enough such that $\partial_M \Pi < 0$, i.e., the token price is falling over time. For this to be the case, issuance must be such that $\partial_M \pi < 0$.

Proof of Proposition 7:

Let $q_t = \frac{M_t - X_t^S}{X_t^S}$ be the number of transactions validated by each speculator. The total transaction fees that accrue to a validator are then $q_t \phi dD_t$. In addition, each validator receives $\frac{\zeta M_t}{X_t^S}$ tokens as seigniorage for each token locked in escrow, x_i^s.

The analysis then proceeds as in the proof of Proposition 1 except now the volatility of user i's state price deflator is now

$$\sigma_{\Lambda_i}(a_i) = \frac{\left(\left(q\phi + \alpha\right)(1 - a_i) + (1 - \phi)\, a_i\right) \pi \hat{A}/M + \left(\mu_P + \frac{1 - a_i}{X_t^S} \zeta M\right) P - rP}{\left(\left(q\phi + \alpha\right)(1 - a_i) + (1 - \phi)\, a_i\right) \sigma_D/M + \sigma_P P}, \tag{IA.31}$$

which is again user i's Sharpe Ratio. This reflects that an adopter receives an expected benefit of $(1 - \phi)\, \pi \hat{A}$ and faces a conditional volatility of $(1 - \phi)\, \sigma_D$. In contrast, speculators who also act as validators receive expected transaction fees $\phi q \pi \hat{A}/M$ and face a conditional volatility of $\phi q \sigma_D$. Validators also receive seigniorage ζ per token. We implicitly assume that shorting as a speculator also shorts a validator's cash flows.

Although seigniorage adds another state variable to the analysis, the total supply of tokens M_t, the growth rate of tokens is deterministic and consequently does not introduce an additional source of risk onto the platform.

The analogue of equation (A.18) for the dual HJB equation for F_i is

$$\begin{aligned}0 &= \frac{1}{\gamma}\log\gamma - \frac{1}{\gamma}\log\Lambda_i + \pi\kappa_i a_i - \partial_{\Lambda_i}F_i\Lambda_i\left(r-\rho\right) - \partial_\Sigma F_i\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 + \frac{1}{2}\partial_{\Lambda_i\Lambda_i}F_i\Lambda_i^2\sigma_{\Lambda_i}^2 \\ &\quad + \frac{1}{2}\partial_{\hat{A}\hat{A}}F_i\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 - \sigma_{\Lambda_i}\left(-\Lambda_i\partial_{\Lambda_i}F_i\sigma_{\Lambda_i} + \partial_{\hat{A}}F_i\frac{\pi\Sigma}{\sigma_D}\right) + \partial_M F\zeta M - rF_i. \quad \text{(IA.32)}\end{aligned}$$

Conjecturing

$$F_i = f_0 + f_{\Lambda_i}\log\Lambda_i + f_i\left(A,\Sigma,M\right), \quad \text{(IA.33)}$$

we arrive at

$$\begin{aligned}f_{\Lambda_i} &= -\frac{1}{r\gamma}, \\ f_0 &= \frac{1}{r\gamma}\left(\log\gamma + \frac{r-\rho}{r}\right), \quad \text{(IA.34)}\end{aligned}$$

where f_i now solves

$$0 = \pi\kappa_i a_i - \frac{1}{2}\frac{1}{r\gamma}\sigma_{\Lambda_i}^2 - \partial_{\hat{A}}f_i\sigma_{\Lambda_i}\frac{\pi\Sigma}{\sigma_D} + \partial_M f_i\zeta M - \partial_\Sigma f_i\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 + \frac{1}{2}\partial_{\hat{A}\hat{A}}f_i\left(\frac{\pi\Sigma}{\sigma_D}\right)^2 - rf_i. \quad \text{(IA.35)}$$

Consequently, the optimal policies are the same as in Proposition 1 except for the modified Sharpe Ratio for validators.

Online Appendix B: Alternative Convenience Yield

In this Appendix, we consider an alternative specification for the convenience yield. Instead of representing a monetary gain for users, the convenience yield provides a mean-variance flow utility benefit to a user $e^{-\rho t}dv\left(\frac{\omega_{it}W_{it}}{P_t}, D_t, a_i\right)$, where

$$
\begin{aligned}
& dv\left(\frac{\omega_{it}W_{it}}{P_t}, D_t, a_i\right) \\
= \quad & \left(a_i + \alpha\left(1 - a_i\right)\right)\frac{\omega_{it}W_{it}}{P_t}dD_t - \frac{1}{2}\left(\left(a_i + \alpha\left(1 - a_i\right)\right)\frac{\omega_{it}W_{it}}{P_t}\right)^2 d\left\langle D\right\rangle_t \\
= \quad & \left(a_i + \alpha\left(1 - a_i\right)\right)\frac{\omega_{it}W_{it}}{P_t}dD_t - \frac{\sigma_D^2}{2}\left(\left(a_i + \alpha\left(1 - a_i\right)\right)\frac{\omega_{it}W_{it}}{P_t}\right)^2 dt, \qquad \text{(IA.36)}
\end{aligned}
$$

$\frac{\omega_{it}W_{it}}{P_t}$ is the number of tokens user i purchases and $d\left\langle D\right\rangle_t$ is the quadratic variation of D_t. Although user preferences are no longer generically time-consistent because of the mean-variance specification, our convex dual approach using to characterizing optimal consumption, investment, and adoption policies do not require it.

Under this specification, the analogue of problem (A.4) in the proof of Proposition 1 is

$$
\begin{aligned}
U_0^i = \sup\nolimits_{c_i,\omega_i,a_i}\inf\nolimits_{\Lambda_{it}} \quad & \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(u\left(c_{it}\right)dt + dv\left(\frac{\omega_{it}W_{it}}{P_t}, D_t, a_i\right)\right)\right] + \Lambda_{i0}W_{i0} \\
& - \lim_{T\to\infty}\mathbb{E}\left[e^{-\rho T}\Lambda_{iT}W_{iT}\right] + \mathbb{E}\left[\int_0^\infty e^{-\rho t}\Lambda_{it}W_{it}\left(\mu_{\Lambda t} + r - \rho\right)dt\right] \\
& + \mathbb{E}\left[\int_0^\infty e^{-\rho t}\Lambda_{it}W_{it}\left(\omega_{it}\left(\mu_{Pt} - r - \sigma_{\Lambda t}\sigma_{Pt}\right) + \pi_t\kappa_i a_{it} + c_{it}\right)dt\right].
\end{aligned}
\qquad \text{(IA.37)}
$$

Assuming the Minimax Theorem holds, the FOCs for ω_{it} and W_{it} imply with some manipulation that again

$$
\mu_\Lambda = -\left(r - \rho\right), \qquad \text{(IA.38)}
$$

and now

$$\left(a_i+\alpha\left(1-a_i\right)\right)\pi\hat{A}-\omega_i\frac{W_i}{P}\left(a_i+\alpha\left(1-a_i\right)\right)^2\sigma_D^2-\Lambda_i\sigma_\Lambda\sigma_P P+\Lambda_i\left(\mu_P P-rP\right)=0. \tag{IA.39}$$

Matching the diffusion terms between optimally invested wealth F_{it} by applying Itô's Lemma and the actual law of motion of wealth W_{it} given user i's trading behavior, it follows that

$$\omega_i W_i \sigma_P = -\partial_\Lambda F \Lambda_i \sigma_\Lambda + \partial_A F \frac{\pi\Sigma}{\sigma_D}. \tag{IA.40}$$

Matching equations (IA.39) and (IA.40), and defining $x_i = \omega_i \frac{W_i}{P}$, we arrive at

$$\sigma_\Lambda = \frac{\left(a_i+\alpha\left(1-a_i\right)\right)\pi\hat{A}+\Lambda_i\left(\mu_P P-rP\right)-\partial_A F\frac{\pi\Sigma}{\sigma_D}\left(a_i+\alpha\left(1-a_i\right)\right)^2\sigma_D^2}{-\partial_\Lambda F\Lambda_i\left(a_i+\alpha\left(1-a_i\right)\right)^2\sigma_D^2+\Lambda_i\sigma_P P}, \tag{IA.41}$$

and

$$x_i = -\partial_\Lambda F\frac{1}{\sigma_P P}\frac{\left(a_i+\alpha\left(1-a_i\right)\right)\pi\hat{A}+\Lambda_i\left(\mu_P P-rP\right)}{-\partial_\Lambda F\left(a_i+\alpha\left(1-a_i\right)\right)^2\sigma_D^2+\sigma_P P}+\frac{\partial_A F\frac{\pi\Sigma}{\sigma_D}}{-\partial_\Lambda F\left(a_i+\alpha\left(1-a_i\right)\right)^2\sigma_D^2+\sigma_P P}. \tag{IA.42}$$

Notice that the expected convenience yield, $\pi\hat{A}$, still governs the return from adoption while the local variance σ_D^2 governs the risk. In the absence of uncertainty about A (i.e., $\Sigma = 0$), optimal token demand reduces to

$$x_i = \frac{\left(a_i+\alpha\left(1-a_i\right)\right)\pi\hat{A}-\Lambda_i rP}{\left(a_i+\alpha\left(1-a_i\right)\right)^2\sigma_D^2}. \tag{IA.43}$$

Since the state price deflator of user i, Λ_i, does not cancel in its diffusion, σ_Λ, or in the optimal token demand, x_i, there are now wealth effects in user i's problem because Λ_i is inversely related (as the costate) to wealth W_i. As a result, characterization of the optimal adoption policy, a_i, and the equilibrium price is significantly less tractable and requires keeping track of the whole dual wealth distribution.

Online Appendix C: Numerical Appendix

In this Appendix, we provide additional details regarding the numerical solution. We can find a numerical solution to characterize this equilibrium from Proposition 2 by solving the system of partial differential equations presented by equations (13), (14) and (19). We use a finite difference method to solve for the functions $f_i(\hat{A}, \Sigma)$, $P(\hat{A}, \Sigma)$ and $\kappa^*(\hat{A}, \Sigma)$ on a uniformly spaced, two-dimensional grid of $(\hat{A}, \Sigma)$. A grid of log-normal distributed κ_i from $[0, \bar{\kappa}]$ is used for numerical integration and as a tool for solving κ^*, as described below. The number of points in the κ_i grid also dictates the number of $f_i(\hat{A}, \Sigma)$ functions in the system (i.e., each user κ_i has a $f_i(\hat{A}, \Sigma)$).

We first make an initial guess $(P^0, \kappa^{*0}, f_i^0)$, then iteratively update $(P^n, \kappa^{*n}, f_i^n)$ until the difference between iterations is sufficiently small. The steady-state solution, as described by Proposition 4, is provided to the program as an initial guess. We then update $P^1(\hat{A}, \Sigma)$ using the "implicit" method, as referred to by other heterogeneous user models like Achdou et al. (2022). In this approach, the next $P^{n+1}(\hat{A}, \Sigma)$ is such that:

$$\frac{P^{n+1} - P^n}{\Delta} + rP^{n+1} = u + \left(\frac{1}{2}\partial_{AA}P^{n+1} - \partial_{\Sigma}P^{n+1}\right)\left(\frac{\pi\Sigma}{\sigma_D}\right)^2$$

where Δ is the step size parameter, π is calculated as $G(\kappa^*)$, the cumulative distribution function of κ, and $u = \left(1 - (1-\alpha)\, v \frac{1-\pi}{\left(\alpha\sigma_D + \partial_{\hat{A}}P\frac{\pi\Sigma}{\sigma_D}\right)^2}\right)\pi\hat{A} + r\gamma v \int_0^{\bar{\kappa}} \left(\frac{\mathbf{1}_{\{\kappa_i \leq \kappa^*\}}}{\frac{\sigma_D^2}{\pi\Sigma} + \partial_{\hat{A}}P} + \frac{\mathbf{1}_{\{\kappa_i > \kappa^*\}}}{\frac{\alpha\sigma_D^2}{\pi\Sigma} + \partial_{\hat{A}}P}\right)\partial_{\hat{A}}f_i dG\left(\kappa_i\right) - r\gamma v$.

This is rearranged into the linear system:

$$\frac{1}{\Delta}(\mathbf{P}^{n+1} - \mathbf{P}^n) + r\mathbf{P}^{n+1} = \mathbf{u}^n + \mathbf{A}^n\mathbf{P}^{n+1}$$

$$\Rightarrow \left(\left(r + \frac{1}{\Delta} \right) \mathbf{I} - \mathbf{A}^n \right) \mathbf{P}^{n+1} = \mathbf{u}^n + \frac{1}{\Delta} \mathbf{P}^n$$

where $\mathbf{A}^n$ is a sparse matrix that takes finite differences. This yields a solution P^{n+1}. Each iteration n also uses a similar method to update f_i^{n+1} for each κ_i user.

Given a system of $P(\hat{A}, \Sigma)$ and $f_i(\hat{A}, \Sigma)$, we can find the cutoff user κ^*. Our method is to: guess each κ_i within our grid as the candidate κ^*, calculate the corresponding π_i, and evaluate equation (14). To rule out multiple equilibria, we pick the highest κ^* that still satisfies equation (14). Each candidate κ_i implies a candidate κ^*. Within each iteration, n, nests another iterative process that updates κ^* until equation (14) yields a fixed point and takes that solution as κ^{*n+1}. Each iteration n updates $(P^n, \kappa^{*n}, f_i^n)$ until the difference between iterations is sufficiently small, and the system of equations converges.