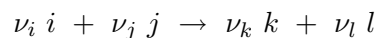


# Reaction Kinetics: Elementary Ideas

25th February 2010

## 1 Rate Laws

Returning to our hypothetical model reaction:



In practice, the rate of reaction is treated as follows, thus introducing the concept of the *rate constant*,  $k$ :

$$R = k \prod_{i=1}^{N_{\text{reactants}}} [i]^{\gamma_i}$$

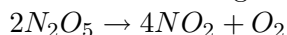
definition of variables:

- $k$  = rate constant (sometimes called rate coefficient; can be determined experimentally; can be estimated theoretically–transition state theory)
- $[i]$  = concentration of REACTANT species "i"
- $\gamma_i$  = Reaction order of REACTANT species "i"
- At this time, we are **NOT** associating the species reactant order  $\gamma_i$  to stoichiometric coefficient!
- $p = \sum_i \gamma_i$  = overall reaction order

NOTE: at this point, the experimental rate equation is written with species reaction orders that have **NO** inherent relation to the stoichiometric coefficients ( $\nu_i, \nu_j, \nu_k, \text{etc}$ ) of the chemical reaction of interest!

## 2 Elementary Reactions

Consider the following overall reaction:



The predicted rate law for this reaction (which happens to be in good agreement with experiment) is:

$$rate = \frac{k_1 k_2}{k_{-1} + 2k_2} [N_2O_5] = k_{effective} [N_2O_5] \quad (1)$$

We observe that the overall rate is **NOT** second order in  $[N_2O_5]$  as one might quickly jump to conclude. This is because the **mechanism** underlying this overall chemical transformation is **complex**; that is, there are several mechanistic steps involved, with certain intermediates involved as well. We will say more about *reaction mechanisms* later. For the present, we mention them in order to emphasize that the stoichiometric coefficients of the chemical reaction are tied to the reaction order **only for elementary reactions or reaction steps**.

## 3 Integrated Rate Equations and Analysis for Elementary Reactions

### I. Zero'th Order Reactions (rare)

The rate of reaction is independent of instantaneous reactant concentration:



$$Rate = k = -\frac{d[A]}{dt} \quad (2)$$

$$[A]_t = [A]_o - kt \quad (3)$$

The rate constant,  $k$ , has units of (concentration/time). The reaction half-life,  $t_{1/2}$ , is the time necessary for one-half of the reactant species to be consumed is given by:

$$t_{1/2} = \frac{[A]_o}{2k} \quad (4)$$

The units of the rate constant  $k$  are in concentration per unit of time ( [moles/liter]/sec ).

## II. First-Order Reactions

A → products

The reaction rate for a First-Order reaction is proportional to the *first power* of the concentration of the reacting species (order=1).

$$\text{Rate} = k[A]^1 = -\frac{d[A]}{dt} \quad (5)$$

Integrating this straightforwardly, keeping in mind the *initial concentration of A*,  $[A]_o$  is defined (boundary condition for solving differential equation):

$$[A]_t = [A]_o e^{-kt} \quad (6)$$

In order to determine if a reaction is First order, using experimental data, we can manipulate the exponential form just determined into a linear form to make analysis easier.

Taking the natural logarithm (natural log) of both sides to arrive at:

$$\ln [A]_t = \ln [A]_o - kt \quad (7)$$

(show plot of linear form here)

The half-life of such a reaction is given by:

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k} \quad (8)$$

Using the last relationship, one can reciprocally determine the rate constant knowing the half-life of a reaction or other physicochemical process.

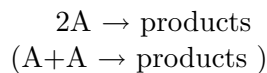
Examples: 1. decay of radioactive nuclei (carbon dating) 2. fluorescence decay of *electronically* excited molecules 3. chemical reactions (as we'll see

later) 4. others

### III. Second Order Reactions

The rate of a second-order reaction is overall 2, and depends on either a single reactant species (to order 2) or on two different reactant species, each with order 1 dependence (recall the definition of reaction order as the sum of the individual species reaction orders). Let's consider the two cases individually.

#### a). Second order in one reactant



$$\text{Rate} = k [A]^2 = -\frac{1}{2} \frac{d[A]}{dt} \quad (9)$$

Performing the integration:

$$\frac{1}{[A]} = \frac{1}{[A]_o} + 2kt \quad (10)$$

Be careful of the factor of  $\frac{1}{2}$  often easily overlooked in the treatment of this reaction.

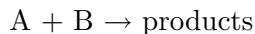
(show figure of  $1/\text{conc}A$  versus  $t$  : straight line)

The half-life for this reaction is given by:

$$t_{\frac{1}{2}} = \frac{1}{2 k [A]_o} \quad (11)$$

#### Second order reaction in two reactant species

If the reaction is given by:



the reaction rate is given by:

$$\text{Rate} = k [A]^1 [B]^1 = -\frac{d[A]}{dt} \quad (12)$$

We obtain for the integrated rate equation:

$$kt = \frac{1}{([A]_o - [B]_o)} \ln \frac{[A][B]_o}{[A]_o[B]} \quad [A]_o \neq [B]_o \quad (13)$$

We can now consider two special cases:

- $[A]_o = [B]_o$
- Because of the one-to-one stoichiometry, we can say for all time  $[A]=[B]$
- The integrated rate law is like that for  $2^{nd}$  order in one component
- $[B]_o \ll [A]_o$  : we can consider reactant A in excess (it essentially remains **constant**)

$$[B] = [B]_o e^{-k't} \quad (14)$$

- $k' = k[A]_o$
- This is pseudo-1st order (species B undergoing transformation in a "sea of A")

## 4 Determining Reaction Orders

In order to determine reaction orders, we can play some games with initial concentrations, excesses of species, etc. In order to do this, we need to measure concentrations (or amounts) as a function of time.

- Quench the reaction, measure concentrations
- For gases, measure pressure versus time
- Spectroscopically follow reactant/product species

## 4.1 Analyzing data

### 4.1.1 Reactions with 1 reactant

A → products

a. Plot or analyze the time behavior of the concentration;

A vs. time

- $\ln[A]$  vs. time
- $1/[A]$  vs. time
- plot to see which dependence give "straight" line

b. Half-life method: measure the relation between  $t_{\frac{1}{2}}$  and  $[A]_o$

- 1<sup>st</sup> order →  $t_{\frac{1}{2}} \propto [A]_o^0$
- 2<sup>nd</sup> order →  $t_{\frac{1}{2}} \propto [A]_o^{-1}$

c. Multiple Life-times ( $t_{\frac{3}{4}}$  and  $t_{\frac{1}{2}}$ ) (**at**  $t_{\frac{3}{4}}$ ,  $[A] = 0.25[A]_o$ )

- 1st order →  $t_{\frac{3}{4}} = \frac{2\ln 2}{k} \rightarrow \frac{t_{\frac{3}{4}}}{t_{\frac{1}{2}}} = 2$
- 2nd order →  $t_{\frac{3}{4}} = \frac{3}{[A]_o k} \rightarrow \frac{t_{\frac{3}{4}}}{t_{\frac{1}{2}}} = 3$

### 4.1.2 Reactions with more than 1 reactant

A + B + C → products

a. Initial Rate Method (vary initial concentrations of a species and measure initial rates)

- For  $[A]_o \frac{\Delta[A]}{\Delta t}|_{t=0} = R_o \equiv k[A]_o^\alpha[B]_o^\beta[C]_o^\gamma$
- For  $[A]_o' \frac{\Delta[A]'}{\Delta t}|_{t=0} = R_o' \equiv k[A]_o'^\alpha[B]_o^\beta[C]_o^\gamma$
- Experimentally determine:  $\frac{R_o}{R_o'} = \left(\frac{[A]_o}{[A]_o'}\right)^\alpha$
- Consider the following cases for the scenario:  $[A]_o' = 0.5[A]_o$ 
  - if  $\frac{R_o}{R_o'} = 1 \rightarrow \alpha = 0$
  - if  $\frac{R_o}{R_o'} = \sqrt{2} \rightarrow \alpha = \frac{1}{2}$
  - if  $\frac{R_o}{R_o'} = 2 \rightarrow \alpha = 1$
  - if  $\frac{R_o}{R_o'} = 4 \rightarrow \alpha = 2$

**b. Flooding or Isolation**

- Let  $[A]_o \ll [B]_o, [C]_o$  (flood system with B and C)
- Then  $[B] \equiv [B]_o$  and  $[C] \equiv [C]_o$
- Thus

$$-\frac{d[A]}{dt} \equiv k'[A]^\alpha \quad (15)$$

- $k' = k[B]_o^\beta [C]_o^\gamma$
- the reaction is made to be pseudo- $\alpha$  order with one reactant