

Physical Chemistry

Lecture 1
Distributions and averages

Characteristics of macroscopic materials

- ◆ Large numbers of particles
- ◆ Distribution of properties
 - Example: speed
 - Describe with a distribution function, $F(v)$
 - ◆ Normalized: $\int_0^{\infty} F(v)dv = 1$
 - ◆ Average of a function of speed

$$\langle f(v) \rangle = \int_0^{\infty} F(v)f(v)dv$$

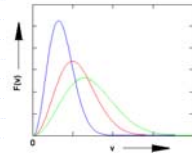
Boltzmann's distribution

- ◆ Speed distribution function, $F(v)$
- ◆ Consider only kinetic energy

$$F(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

- ◆ This is normalized

$$\int_0^{\infty} F(v)dv = 1$$



Calculating average molecular properties

- ◆ Use the distribution function

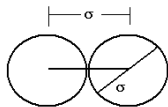
$$\overline{f(v)} = \int_0^{\infty} f(v)F(v)dv$$

- ◆ Example: average speed

$$\bar{v} = \int_0^{\infty} vF(v)dv = \sqrt{\frac{8kT}{\pi m}}$$

Molecular collisions

- ◆ Collisions dominate effects that depend on close proximity of molecules
- ◆ "Hard-sphere" model

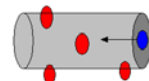


Collisions of a molecule

- ◆ Determined by the number of molecules in the space ($N = n \cdot V$)

- ◆ Collision frequency

$$z_{AA} = n^* \pi \sigma^2 \bar{v}_{rel} = \sqrt{2} n^* \pi \sigma^2 \bar{v}$$



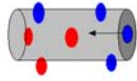
- ◆ Mean-free path

$$\lambda = \frac{\bar{v}}{z_{AA}} = \frac{1}{\sqrt{2} \pi \sigma^2 n^*} = \frac{kT}{\sqrt{2} P \pi \sigma^2}$$

Collisions of unlike molecules

- ◆ Three different kinds of collision
- ◆ Collision frequencies per molecule

$$\begin{aligned}\bar{z}_{AA} &= n_A^* \pi \sigma^2 \bar{v}_{rel} \\ &= \sqrt{2} n_A^* \pi \sigma^2 \bar{v}\end{aligned}$$



$$\begin{aligned}\bar{z}_{A:B} &= \pi \sigma_{AB}^2 \bar{v}_{AB} n_B^* \\ \bar{z}_{B:A} &= \pi \sigma_{AB}^2 \bar{v}_{AB} n_A^*\end{aligned}$$

$$\sigma_{AB} = \frac{\sigma_A + \sigma_B}{2}$$

Total collisions in a mixture

- ◆ Often must know the total number of collisions per unit time

$$Z_{AA} = \bar{z}_A \frac{n_A^*}{2} = \frac{\pi \sigma_A^2 \bar{v}_A}{\sqrt{2}} n_A^{*2}$$

$$Z_{AB} = \bar{z}_{A:B} n_A^* = \pi \sigma_{AB}^2 \bar{v}_{AB} n_A^* n_B^*$$

$$Z_{AB} = \bar{z}_{B:A} n_B^* = \pi \sigma_{AB}^2 \bar{v}_{AB} n_A^* n_B^*$$

Summary

- ◆ Simple kinetic theory allows calculation of various average properties
- ◆ Calculate average properties with the distribution function
- ◆ Collision frequency important for properties that depend on the molecules being in close proximity
- ◆ Mixtures have multiple kinds of collisions