1^{st} "Law": $\quad dU = đq + đw$

dU: state function (independent of path)

$đq$, $đw$: not state functions (depend on path)

U can be written as a function of state variables such as T, V

Let's consider 1^{st} "Law":

$$dU = đq + đw$$

$$dU = C(T)\,dT + (-p_{ext}\,dV)$$

$$dU = C(T)\,dT - p_{ext}\,dV$$

For a <u>reversible</u> process, $p_{ext} = p$ where p is the pressure associated with the EOS for a fluid. Recall that EOS describes <u>Equilibrium</u> <u>states</u> of a fluid.

$\therefore$ p comes from:

1) I. G. EOS $\quad pV = nRT$

or

2) Van der Waals EOS $\quad p = \dfrac{nRT}{V-b} - \dfrac{an^2}{V^2}$

Note: $\text{đ}w = -P_{ext}\, dV$

For reversible process: $P_{ext} = P$

$\therefore \quad \text{đ}w = -P\, dV$

I. G. EOS for P: $\text{đ}w = -\dfrac{nRT}{V}\, dV$

$$\text{đ}w = -nRT\, \frac{dV}{V}$$

(*)

$$\text{đ}w = -nRT\, d(\ln V)$$

make sure you understand (*)

→ it's like this: $\dfrac{d(\ln x)}{dx} = \dfrac{1}{x}$

$$\therefore \quad d(\ln x) = dx\, \frac{1}{x} = \frac{dx}{x}$$

$$\therefore \quad \frac{dx}{x} = d(\ln x)$$

or

$$\frac{dV}{V} = d(\ln V)$$

this differential form arises often!! memorize it!!!

Back to $đw = -\frac{nRT}{V} dV = -nRT\, d(\ln V)$

I.G. EOS:

for Iso thermal change in volume, $V_1 \rightarrow V_2$

$\Longrightarrow$ $$W = \int_{V_1}^{V_2} đw = \int_{V_1}^{V_2} -nRT\, d(\ln V) = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

make sure you Know this and how we arrived at this equation!!!!

Consider this:

if for a fluid $dU = C_V\, dT$ (where C_V is constant)

we can use the 1st "Law" as:

$$dU = đq + đw$$

if we think about a reversible, adiabatic process:
$(P_{ext} = P)$, $(đq = 0)$

$$dU = đw_{rev} = -p\,dV$$

$$\therefore \quad dU = -p\,dV$$

But we are told for this fluid,

$$du = C_v \, dT$$

So: $$C_v \, dT = -p \, dV$$

If p is given by I.G. EOS

$$p = \frac{nRT}{V}$$

we can write:

$$C_v \, dT = -nRT \frac{dV}{V}$$

divide both sides by T

$$C_v \frac{dT}{T} = -nR \frac{dV}{V}$$

C_v, n, R are constants

So, integrate from T_1 to T_2 and V_1 to V_2

$$C_v \ln\left(T_2/T_1\right) = -nR \ln\left(V_2/V_1\right)$$

$$\ln\left(T_2/T_1\right)^{C_v} = \ln\left(V_2/V_1\right)^{-nR}$$

use I.G. EOS.

$$\ln\left(T_2/T_1\right)^{C_v} = \ln\left(\frac{T_2}{P_2}\frac{P_1}{T_1}\right)^{-nR}$$

$$\therefore \quad \left(\frac{T_2}{T_1}\right)^{C_v} = \left(\frac{T_2}{T_1}\frac{P_1}{P_2}\right)^{-nR}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{T_2}{T_1}\frac{P_1}{P_2}\right)^{\frac{-nR}{C_v}}$$

$$\left(\frac{P_2V_2}{P_1V_1}\right) = \left(\frac{P_2V_2}{P_1V_1}\frac{P_1}{P_2}\right)^{\frac{-nR}{C_v}}$$

$$= \left(\frac{V_2}{V_1}\right)^{\frac{-nR}{C_v}}$$

$$\left(\frac{P_2V_2}{P_1V_1}\right)\left(\frac{V_2}{V_1}\right)^{\frac{nR}{C_v}} = 1$$

$$\frac{P_2V_2^{1+\frac{nR}{C_v}}}{P_1V_1^{1+\frac{nR}{C_v}}} = 1$$

$$\therefore \quad P_2V_2^{1+\frac{nR}{C_v}} = P_1V_1^{1+\frac{nR}{C_v}}$$

For I.G. $C_p - C_v = nR$

$$\therefore \quad 1+\frac{nR}{C_v} = 1+\frac{C_p-C_v}{C_v} = 1+\frac{C_p}{C_v}-1 = \frac{C_p}{C_v}$$

$$\therefore \quad P_2 V_2^{(C_p/C_v)} = P_1 V_1^{(C_p/C_v)}$$

define : $\dfrac{C_p}{C_v} \equiv \gamma$

So : $P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

or in general $\boxed{P V^{\gamma} = \text{constant}}$

For Adiabatic Reversible Process involving Ideal Gas

Understand All steps leading to this equation