Quasi-3D Nearshore Circulation Equations: a CL-Vortex Force Formulation

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We formulate a CL-vortex form of surface wave force for a quasi-3D nearshore circulation model. The CL-vortex force formulation is obtained by applying surface wave equations to depth-integrated and wave-averaged momentum equations. A new splitting algorithm of current velocity is used to facilitate the application of wave equations to the wave-averaged equations. The derivation shows that the CL-vortex term arises from both the wave refraction by current shear and the wave-current interaction shown in the radiation stress type momentum equations. In the vertical direction, the equation governing the vertical structure of current velocity is driven by a surface stress related to wave dissipation. Numerical tests show the CL-vortex formulation performs identically with the radiation stress formulation in modeling of rip currents. However, without fully coupling of wave and current models, the CL-vortex formulation gives a more reasonable result than the radiation stress formulation does.

INTRODUCTION

Recently much attention has been paid to different formulations of surface wave force in wave-driven ocean and coastal circulations (e.g., McWilliams et al., 2004, Mellor, 2003, Smith, 2006 and others). Basically, the analytical expressions for surface wave force and wave-current interaction can be classified into two types. One is the classical wave 'radiation stress' concept presented by Longuet-Higgins and Stewart (1962, 1964) and many others in depth-integrated and short wave-averaged equations. Mellor (2003) recently used the same concept to derive short wave-averaged 3-D equations with a depth-dependent wave-induced force. A direct application of this kind of depth-dependent wave-induced force was conducted by Xia et al. (2004) who related the vertical variation of current to the vertical structure of radiation stresses.

Another type of wave force is the surface wave force initially derived by Garrett (1976) in the study of Langmuir circulation generation. The wave driving forces include the wave dissipation term and the wave-averaged vortex forcing term which has been identified later by Leibovich (1980) and Smith (1980) as the vertically integrated form of the 'CL vortex-force' derived by Craik and Leibovich (1976). Dingemans et al. (1987) also presented a similar formulation of this type of wave driving force, though the current refraction, that may result in the vortex-force term, was recognized to give insignificant contributions under the conditions of slowly varying wave fields. Smith (2006) extended the formulation of Garrett (1976) to include finite-depth effects and provided some insight into physical interpretation of each forcing term in depth-integrated equations. A

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depth-dependent wave force of this type can be found in McWilliams et al. (2004) who showed a series of equations in different time scales for surface waves, infra-gravity waves and low-frequency currents in a coupled system.

Most recently, Newberger and Allen (2006) applied a similar form of the CL-vortex formulation to forcing a three-dimensional hydrostatic primitive-equation model in the surf zone. Their analysis was focused on shallow water dynamics involving interactions of linear waves with wave-averaged mean currents. The short wave forcing in their approach consists of a surface stress and a body force. The surface stress is proportional to the wave energy dissipation which is basically caused by wave breaking in the surf zone. The body force arises from the so-called local radiation stresses. Under an assumption of shallow water currents with linear waves, the body force includes one term that is related to the vortex force and a second term that is related to gradients of part of the radiation stress tensor. The vortex force, which is a product of the mean wave momentum and the vertical component of the depth-averaged mean vorticity vector, is similar to the "refraction force" in Smith (2006) except that Smith's refraction force is evaluated using the mean vorticity at the mean surface. The body force was uniformly applied to the vertical water column in their three-dimensional model.

Although the consistency in the two types of theoretical formulations can be found without difficulties, the numerical models based the different wave forcing formulations may perform differently. Dingemens et al. (1987) pointed out that the formulation in terms of the wave dissipation yields more trustworthy results as the radiation stress tensor can be a rapidly varying function of the spatial coordinates, numerical differentiation can lead to poor results.

Among two- or three-dimensional models of wave-induced nearshore circulation, a quasi-3D nearshore circulation model developed by Svendsen et al. (2004) is a simple approach to 3-D modeling of wave-induced nearshore circulation. The quasi-3D equations reveal three-dimensional dispersion of momentum in wave-induced nearshore currents, wave-current interaction, and the contribution of short wave forcing to a solution of vertical current profile. The objective of this study is to describe the CL-vortex type short wave force formulation in the quasi-3D model frame.

In the present paper, we re-formulate the quasi-3D nearshore circulation equations (SHORECIRC equations, Putrevu and Svendsen, 1999) using a new splitting algorithm of current velocity proposed by Haas et al. (2003). A new type of wave forcing is derived for both the depth-integrated and wave-averaged momentum equations and the equation governing the vertical structure of current velocity. Numerical consistency between two different wave force formulations is discussed in idealized rip current simulations.

DERIVATION

Split of wave-averaged current velocity

Slightly different from the splitting method used by Putrevu and Svendsen (1999), the short wave-averaged current velocity $V_\alpha(z)$ is split into a 'undisturbed' depth-averaged mean current $V_{m\alpha}$ and vertical variation of mean current $V_{d\alpha}(z)$:

$$V_\alpha(z) = V_{m\alpha} + V_{d\alpha}(z) \tag{1}$$

where

$$V_{m\alpha} = \frac{1}{h}\int_{-h_0}^{\bar{\zeta}} \bar{u}_\alpha dz \tag{2}$$

or

$$V_{m\alpha} = \frac{1}{h}\left(\overline{\int_{-h_0}^{\zeta} u_\alpha dz} - \overline{\int_{\bar{\zeta}}^{\zeta} u_\alpha dz}\right) \tag{3}$$

and

$$\int_{-h_0}^{\bar{\zeta}} V_{d\alpha} dz = 0 \tag{4}$$

u_α represents the instantaneous horizontal velocity including the wave component. The first term on the right of (3) is the total volume flux and the second term is the net wave volume flux:

$$Q_{w\alpha} = \overline{\int_{\bar{\zeta}}^{\zeta} u_\alpha dz} \tag{5}$$

Notice that Putrevu and Svendsen (1999) used the total volume flux as a main variable in their split. This splitting method was also used by Haas et al. (2003) who pointed out that this split is more physical and has advantages in the simplification of the 3D dispersive mixing terms.

Equations of depth-integrated and wave-averaged current

Following the derivation of Putrevu and Svendsen (1999), the depth-integrated, short-wave-averaged momentum equations read

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial}{\partial x_\alpha}(V_{m\alpha}h + Q_{w\alpha}) = 0 \tag{6}$$

and

$$\begin{aligned} &\frac{\partial}{\partial t}(V_{m\alpha}h) + \frac{\partial Q_{w\alpha}}{\partial t} + \frac{\partial}{\partial x_\beta}\left[V_{m\alpha}V_{m\beta}h + Q_{w\alpha}V_{m\beta} + V_{m\alpha}Q_{w\beta}\right. \\ &\left. + \overline{\int_{-h_0}^{\zeta} V_{d\alpha}V_{d\beta}dz} + \overline{\int_{\zeta_t}^{\zeta}(u_{w\alpha}V_{d\beta} + V_{d\alpha}u_{w\beta})dz}\ \right] \\ &+ \frac{1}{\rho}\frac{\partial T_{\alpha\beta}}{\partial x_\beta} + gh\frac{\partial \bar{\zeta}}{\partial x_\alpha} - \frac{\tau_\alpha^s}{\rho} + \frac{\tau_\alpha^B}{\rho} + \frac{1}{\rho}\frac{\partial S_{\alpha\beta}}{\partial x_\beta} = 0 \end{aligned} \tag{7}$$

where τ_α^s and τ_α^B represent the surface stress and bottom stress, respectively. In (7) the radiation stress is defined by

$$S_{\alpha\beta} = \overline{\int_{-h_0}^{\zeta}(\rho u_{w\alpha}u_{w\beta} + p\delta_{\alpha\beta})dz} - \frac{1}{2}\rho g h^2 \delta_{\alpha\beta} \tag{8}$$

where p is the total pressure and $\delta_{\alpha\beta}$ is the Kronecker delta function. Similarly to Putrevu and Svendsen (1999), we assume that $V_{d\alpha}$ is approximately constant in the interval $\bar{\zeta}$ to ζ and thus (4) may be simplified as follows

$$\overline{\int_{-h_0}^{\zeta} V_{d\alpha} dz} \approx \int_{-h_0}^{\bar{\zeta}} V_{d\alpha} dz = 0 \tag{9}$$

With the same approximation, the integrals in (7) may be written as

$$\overline{\int_{-h_0}^{\zeta} V_{d\alpha} V_{d\beta} dz} + \overline{\int_{\zeta_t}^{\zeta} (u_{w\alpha} V_{d\beta} + V_{d\alpha} u_{w\beta}) dz} \approx \int_{-h_0}^{\bar{\zeta}} V_{d\alpha} V_{d\beta} dz + V_{d\beta}(\bar{\zeta}) Q_{w\alpha} + V_{d\alpha}(\bar{\zeta}) Q_{w\beta} \tag{10}$$

Using (10), (7) is reorganized as

$$\begin{aligned} &\frac{\partial}{\partial t}(V_{m\alpha} h) + \frac{\partial}{\partial x_\beta}\left[V_{m\alpha} V_{m\beta} h + \int_{-h_0}^{\bar{\zeta}} V_{d\alpha} V_{d\beta} dz\right] \\ &+ \frac{1}{\rho}\frac{\partial T_{\alpha\beta}}{\partial x_\beta} + gh\frac{\partial \bar{\zeta}}{\partial x_\alpha} - \frac{\tau_\alpha^s}{\rho} + \frac{\tau_\alpha^B}{\rho} \\ &+ \frac{\partial Q_{w\alpha}}{\partial t} + \frac{\partial}{\partial x_\beta}\left[\left(V_{m\beta} + V_{d\beta}(\bar{\zeta})\right) Q_{w\alpha} + \left(V_{m\alpha} + V_{d\alpha}(\bar{\zeta})\right) Q_{w\beta}\right] \\ &+ \frac{1}{\rho}\frac{\partial S_{\alpha\beta}}{\partial x_\beta} = 0 \end{aligned} \tag{11}$$

Notice that the wave-current interaction terms in the third line of (11) include the mean current value at the mean surface, i.e., $(V_{m\alpha} + V_{d\alpha}(\bar{\zeta}))$ or $V_\alpha(\bar{\zeta})$, rather than the deviation of the mean current at the surface, as in the equations of Putrevu and Svendsen (1999).

To connect (11) to wave evolution equations, the radiation stresses may be expressed using the form evaluated by Longuet-Higgins and Stewart (1962, 1964), i.e.,

$$S_{\alpha\beta} = \rho Q_{w\alpha} C_\beta^g + \rho h J \delta_{\alpha\beta} = E^w\left(\frac{k_\alpha C_\beta^g}{\sigma}\right) + \rho h J \delta_{\alpha\beta} \tag{12}$$

where E^w is the wave energy, k_α presents the wave number, C_α^g the wave group velocity, σ the intrinsic radian frequency and

$$\rho h J = \frac{1}{2}\rho h(\overline{u_w^2} - \overline{w^2}) = \rho Q_{w\alpha}(C_\alpha^g - \frac{1}{2}C_\alpha) = E^w\left(k\frac{C^g}{\sigma} - \frac{1}{2}\right) \tag{13}$$

Note that the definition of J is slightly different from that in Smith (2006) by a factor of ρ.

Following Smith (2006), we employ the conservation of wave action and wave crests, i.e.,

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_\alpha}\left(A(C_\alpha^g + V_\alpha(\bar{\zeta}))\right) = -D_w \tag{14}$$

and

$$\frac{\partial k_\alpha}{\partial t} + \frac{\partial}{\partial x_\alpha}\left(\sigma + k_\beta V_\beta(\bar{\zeta})\right) = 0 \tag{15}$$

to get a wave momentum evolution equation:

$$\frac{\partial Q_{w\alpha}}{\partial t} + \frac{\partial}{\partial x_\beta}\left[Q_{w\alpha}(C^g_\beta + V_\beta(\bar{\zeta}))\right] = -\frac{1}{\rho}k_\alpha(D_w)$$
$$-Q_{w\beta}\frac{\partial V_\beta(\bar{\zeta})}{\partial x_\alpha} - J\frac{\partial h}{\partial x_\alpha} \tag{16}$$

where D_w represents wave dissipation caused by wave breaking. In derivation of (16), the relation between Q_w and wave action A, i.e.,

$$Q_{w\alpha} = Ak_\alpha/\rho \tag{17}$$

is used. Using (16) to replace the wave-induced terms shown in the third and fourth lines of (11), we get the mean current equations:

$$\frac{\partial}{\partial t}(V_{m\alpha}h) + \frac{\partial}{\partial x_\beta}\left[V_{m\alpha}V_{m\beta}h + \int_{-h_0}^{\bar{\zeta}} V_{d\alpha}V_{d\beta}dz\right]$$
$$+\frac{1}{\rho}\frac{\partial T_{\alpha\beta}}{\partial x_\beta} + gh\frac{\partial\bar{\zeta}}{\partial x_\alpha} - \frac{\tau^s_\alpha}{\rho} + \frac{\tau^B_\alpha}{\rho} - \frac{F_{w\alpha}}{\rho} = 0 \tag{18}$$

where $F_{w\alpha}$ is the new form of wave force:

$$F_{w\alpha} = k_\alpha D_w + \rho Q_{w\beta}\left(\frac{\partial V_\beta(\bar{\zeta})}{\partial x_\alpha} - \frac{\partial V_\alpha(\bar{\zeta})}{\partial x_\beta}\right)$$
$$-\rho V_\alpha(\bar{\zeta})\frac{\partial Q_{w\beta}}{\partial x_\beta} - \rho h\frac{\partial J}{\partial x_\alpha} \tag{19}$$

Or its vector form

$$\vec{F}_w = \vec{k}D_w + \rho\vec{Q}_w \times (\nabla \times \vec{V}(\bar{\zeta}))$$
$$-\rho\vec{V}_\alpha(\bar{\zeta})(\nabla\cdot\vec{Q}_w) - \rho h\nabla(J) \tag{20}$$

It can be seen from the derivation that the vortex term $\vec{Q}_w \times (\nabla \times \vec{V}(\bar{\zeta}))$ arises from both the wave-current interaction term in (11) and wave refraction by current shear in (16). The equivalent wave forcing formulation in the radiation stress form is

$$\vec{F}_w = -\rho\frac{\partial\vec{Q}_w}{\partial t} - \rho\nabla\cdot\left(\vec{V}(\bar{\zeta})\vec{Q}_w + \vec{Q}_w\vec{V}(\bar{\zeta})\right) - \nabla\cdot\vec{S} \tag{21}$$

Without including the wave refraction by current shear (generally shown in wave equations), the radiation stress formulation (21) would not explicitly show the vortex term.

Equation for vertical variation of mean current

Using the same strategy as in Putrevu and Svendsen (1999), we derive the equation governing the vertical structure of $V_{d\alpha}$. By subtracting the mean current momentum from the 3D wave-averaged equations, the lowest order of the equation for vertical variation of mean current can be written as

$$\frac{\partial V_{d\alpha}}{\partial t} - \frac{\partial}{\partial z}(\nu \frac{\partial V_{d\alpha}}{\partial z}) = -\frac{1}{\rho h} F_{w\alpha} - \frac{1}{\rho} f_\alpha - \frac{1}{h} V_{m\alpha} \frac{\partial Q_{w\beta}}{\partial x_\beta} - \frac{1}{\rho h}(\tau_\alpha^s - \tau_\alpha^B) \quad (22)$$

where f_α is local radiation stress defined by

$$f_\alpha = \rho \overline{\frac{\partial}{\partial x_\beta}(u_{w\alpha} u_{w\beta})} + \rho \overline{\frac{\partial w_w u_{w\alpha}}{\partial z}} - \rho \overline{\frac{\partial w_w^2}{\partial x_\alpha}} \quad (23)$$

In shallow water, f_α may be evaluated as (Newberger and Allen, 2006)

$$f_\alpha = -\frac{\rho}{h} Q_{w\beta} \left(\frac{\partial V_\beta(\bar{\zeta})}{\partial x_\alpha} - \frac{\partial V_\alpha(\bar{\zeta})}{\partial x_\beta} \right) + \rho \frac{\partial}{\partial x_\alpha} \left(\frac{J}{h} \right) \quad (24)$$

Applying (24) to (22) yields the equation governing $V_{d\alpha}$ in shallow water:

$$\frac{\partial V_{d\alpha}}{\partial t} - \frac{\partial}{\partial z}(\nu \frac{V_{d\alpha}}{\partial z}) = \frac{1}{\rho h}(-k_\alpha D_w - \tau_\alpha^s + \tau_\alpha^B) \quad (25)$$

(25) can be solved analytically or numerically by applying the surface and bottom boundary conditions given by Putrevu and Svendsen (1999) and the condition (4). It should be mentioned that only leading terms are retained in the derivation of (22) in order to maintain consistency with the SHORECIRC equations.

NUMERICAL TEST

To make a simple test on the numerical consistency between two different wave force formulations, we implemented (20) in the 2D mode of SHORECIRC equations (Svendsen et al., 2004). The lateral mixing caused by vertical variation of current velocity was not taken into account by neglecting (25) and switching off the 3D dispersion terms in (18). An unsteady wave-driver developed by Kennedy and Kirby (2004) was used to provide the circulation model with non-stationary wave forcing. The model coupling system which can switch to two different wave force formulations was used in rip current simulations.

We used an idealized bathymetry (Yu and Slinn, 2003) as shown in Figure 1. A normally incident wave, with a wave period of 10s and a wave height of 1.8 m, was simulated in the wave model. Wave-current interaction can be modeled by coupling the wave model and the circulation model each time step. Similar to the results shown in Yu and Slinn (2003), our results indicate that wave-current interaction is important in the rip current system. The interaction reduces the strength of the currents and restricts their offshore extent. When the wave

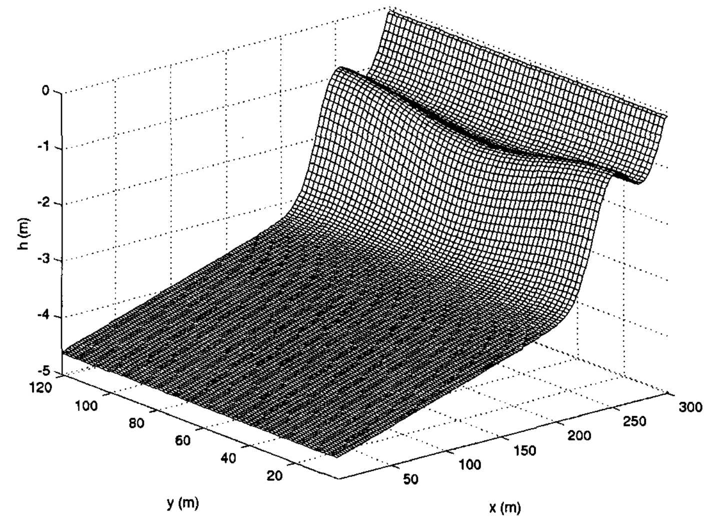

Figure 1. Idealized bathymetry in Yu and Slinn (2003)

model and the circulation model are fully coupled, the two different wave force formulations basically give very similar results as shown in Figure 2. Figure 3 shows the comparison of time-averaged rip current profiles between the radiation stress formulation (dashed line) and the CL-vortex formulation. The term-by-term comparisons of the two wave force formulations also indicate that the two different formulations are basically equivalent in the rip current simulations. In x-direction, the dominant term in (21) is $-\nabla \cdot \vec{S}$ which is shown in Figure 4 (a). The dominant terms in (20) are $\frac{1}{\rho}\vec{k}D_w$ and $-h\nabla(J)$. The sum total of the two terms is shown in Figure 4(b) which is similar to that in Figure 4 (a). In y-direction, the dominant terms in (21) are $-\rho\nabla \cdot \left(\vec{V}(\bar{\zeta})\vec{Q}_w + \vec{Q}_w\vec{V}(\bar{\zeta})\right)$ and $-\nabla \cdot \vec{S}$. The sum total of the two terms is shown in Figure 4 (c). The dominant term in (20) is $\rho\vec{Q}_w \times (\nabla \times \vec{V}(\bar{\zeta}))$ and is shown in Figure 4 (d). The comparison between Figures 4 (c) and 4 (d) clearly shows that the effect of the vortex type force seems to occur in the integration of the two dominant terms in (21), that is similar to the CL-vortex force in (20).

Without wave-current interaction, however, the two wave force formulations perform very differently in the rip current simulations. The effect of vortex type force does not show up in the radiation stress formulation (21). Figure 5 (a) shows the sum value of the two dominant terms in (21) in y-direction. It differs from the value calculated from the CL-vortex force and shown in Figure 5 (b). Figure 6 shows a comparison between the time-averaged rip current profiles from the

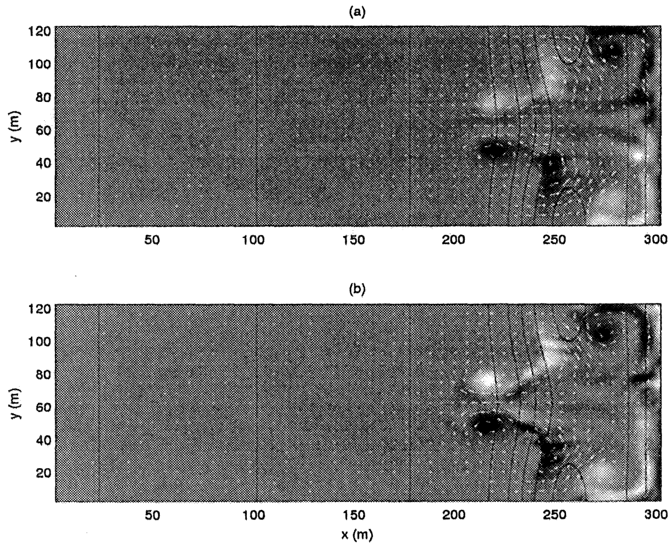

Figure 2. Snapshot of current and vorticity (color) field: (a)radiation stress formulation (b) CL-vortex formulation.

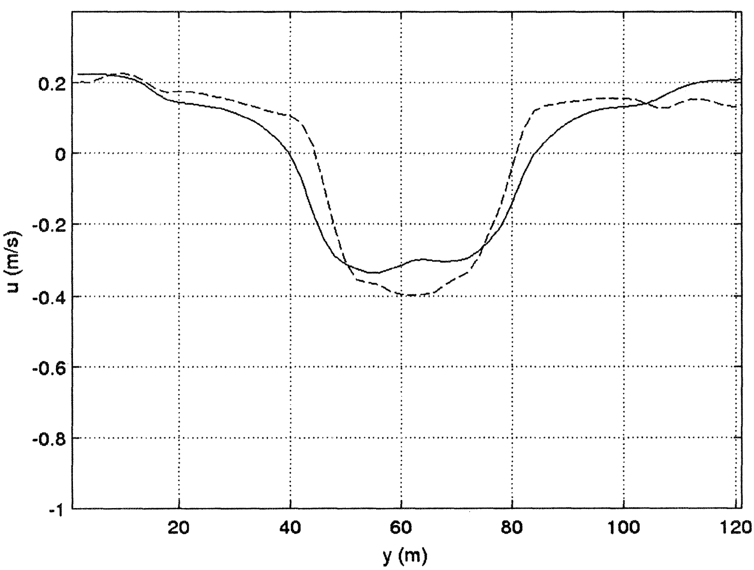

Figure 3. Comparison of time-averaged rip current profiles at x = 220 m with wave-current interaction (dashed line: radiation stress formulation, solid line: CL-vortex formulation).

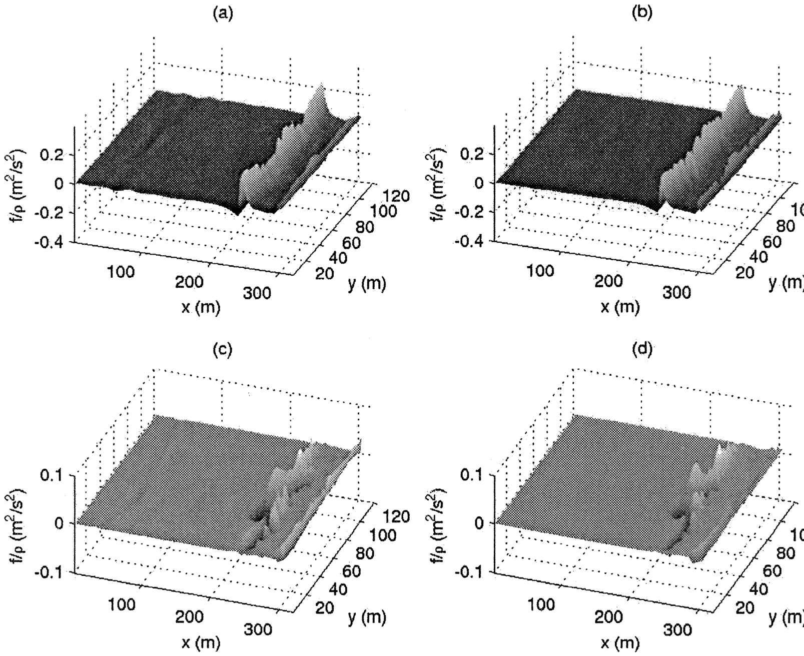

Figure 4. Comparisons of wave forcing when wave and current are fully coupled (a) radiation stress formulation in x direction (b) CL-vortex formulation in x direction (c) radiation stress formulation in y direction (d) CL-vortex formulation in y direction

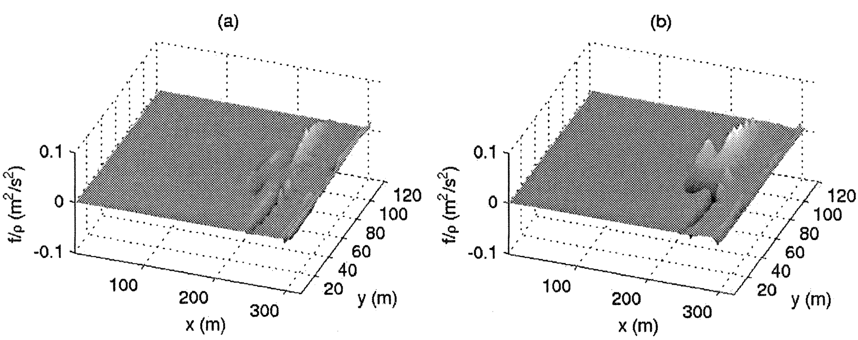

Figure 5. Comparisons of wave forcing when wave and current are not coupled (a) radiation stress formulation in y direction (b) CL-vortex formulation in y direction

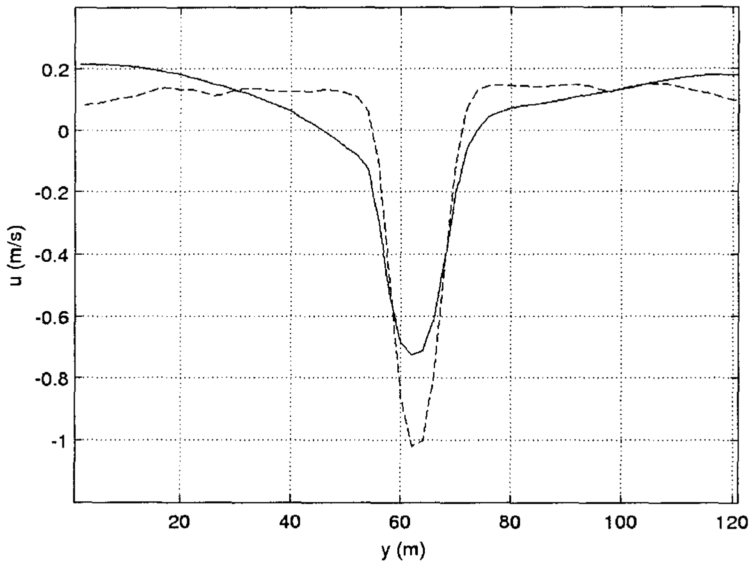

Figure 6. Comparison of time-averaged rip current profiles at x = 220 m without wave-current interaction (dashed line: radiation stress formulation, solid line: CL-vortex formulation)

radiation stress formulation (dashed line) and the CL-vortex formulation. The CL-vortex formulation predicts a weaker rip current than the radiation stress formulation because the CL-vortex forcing plays an role in widening the rip current neck and thus weakening the strength of the rip current.

CONCLUSIONS

In this study, we described a CL-vortex wave force for a quasi-3D nearshore circulation model. A new splitting algorithm of current velocity presented in Haas et al. (2003) is applied to keep aligned with Smith's (2006) derivation of the CL-vortex force in general application of wave-induced circulations. Our derivation shows that the wave force driving the depth-integrated and wave-averaged momentum equations is the same as in Smith's derivation with finite-depth effects included. The new wave formulation includes a wave dissipation term, a CL-vortex term, a correction term due to mass-flux lost on non-zero current velocity at surface, and a term described as a hydrostatic pressure gradient in Smith (2006). In the equation governing the vertical structure $V_{d\alpha}$, the force from the short wave contribution only consists of a wave dissipation term.

It is interesting to compare our formulation to the short wave force derived by Newberger and Allen (2006) in application of a three-dimensional circulation model in the surf zone. The short wave force in their approach includes a surface stress caused by wave dissipation and a body force which consists of the

CL-vortex force and an extra force related to gradients of part of the radiation stresses. In our application to Quasi-3D equations, both the surface stress and the body stress (integrated) are included in the depth-integrated and wave-averaged momentum equations. In the equation governing the vertical structure of current velocity, the surface stress is the only force from the short wave contribution. Our results are theoretically consistent with Newberger and Allen (2006).

The numerical consistency in using the different types of wave forces was discussed in the paper. Simulations of rip current with a idealized bathymetry show that the two formulations perform identically when wave-current interaction is included in model coupling. However, if leaving out wave-current interaction, the CL-vortex formulation gives a result closer to the result with the wave-current interaction than the radiation stress formulation does. The vortex forcing is significant in the rip current case with a fast-varying wave field. It plays an important role in widening the rip current neck and thus weakening the strength of rip current.

Our further study may include several developments and applications of the new wave formulations, such as, 1) implementation of the full set of equations, i.e., (18) and (25), to take into account 3D dispersion effects; 2) quantitative comparison between models with different wave force formulations; 3) evaluation of model results using measurement data; 4) optimization of numerical time step used in wave-current model coupling; 5) model performance in predictions of various nearshore phenomena such as longshore currents, infra-gravity waves, and shear waves.

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