Wave-averaged and Wave-resolving Numerical Modeling of Vorticity Transport in the Nearshore Region: the SANDYDUCK Case Study

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The paper analyzes the mixing features in numerically generated shear waves by simulating the circulation during SANDYDUCK experiments. Different wave resolving Boussinesq type models and a wave averaged model have been compared. They produce different velocity/vorticity fields and in turn different mixing properties. Lagrangian statistics have been used to study this aspect. Differences in absolute and relative dispersion and diffusion have been pointed out. All the models seem to follow a Richardson-like intermediate regime for the longshore relative dispersion even though some results seem to suggest a dependency from particles separation weaker than the classical $4/3$ law.

INTRODUCTION

Numerical modeling of unstable nearshore flows has become quite popular in the last decade. A class of models, usually adopted for this purpose, relies on averaging the equations over the time scales of the short waves (hence wave averaged models). The wave characteristics, and in turn the forcing terms are provided by another model (wave driver). Examples of longshore current studies using these models are Özkan Haller and Kirby (1999), Zhao et al. (2003), among others. On the other hand in wave resolving models, all the time scales of the hydrodynamics are explicitly solved. Boussinesq-type models (BTMs hereinafter) belong to this class of models and have been used, for instance, in Chen et al. (2003), Gobbi et al. (2000) and Johnson and Pattiaratchi (2006) among others. When applied in similar conditions, the two approaches may produce different results in terms of flow description due to different forcing mechanisms. Differences may arise within the same class of models that share a similar set of equations. In particular it has been shown in Gobbi et al. (2000) that different transport equations for vertical vorticity may be written starting from different BTMs, depending on the order of magnitude of the retained vertical vorticity. The modeled flows in wave averaged models are characterized by less energetic fluctuations with respect to those predicted by BTMs. In turn the vorticity transport is radically different,

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producing different mixing features. An interesting approach which allows to investigate this latter aspect, is the study of the trajectories of particles (tracers) released in the fluid, as done, for example in oceanic field experiments (e.g. LaCasce and Bower, 2000). Following the theory of turbulence, emphasis is given to Lagrangian statistics rather than on the description of the single drifter trajectory. The statistics allow to investigate the regimes of the absolute and relative dispersion of the floaters. The outcome of both Johnson and Pattiaratchi (2004) and Piattella et al. (2006) is that relative dispersion depends from particles separation with a $4/3$ power law both within and outside the surf zone. The present study aims at identifying the characteristics of the dispersion induced by the unstable longshore currents as modeled by both wave resolving BTMs and a wave averaged model and investigate the reasons of these differences.

NUMERICAL MODELS USED

Wave resolving models

The wave resolving model used in this study is based on the Boussinesq type equations described also in Gobbi et al. (2000). The equations are defined in a cartesian coordinate system x, y, z, with z directed upward. We adopt the classical non-dimensionalization of the governing equations choosing the offshore short waves amplitude a_0, water depth h_0 and inverse of the wave number $1/k_0$ as characteristic length scales for wave amplitude, water depth and wave length, leading to dimensionless parameters $\delta = a_0/h_0$ and $\mu = k_0 h_0$. The equations are obtained without any limitation on δ. Terms only up to $O(\mu^2)$ are retained. The velocity is defined introducing $\mathbf{u}_\alpha = (u_\alpha, v_\alpha)$ as the horizontal velocity vector at an arbitrary elevation z_α. The integration over depth of the Euler equation leads to a dimensionless set of governing equations in which the mass conservation equation is given by:

$$\eta_t + \nabla \cdot \mathbf{M} = O(\mu^4) \tag{1}$$

where M is the horizontal volume flux vector, η is the free surface. The momentum conservation equation is given by:

$$\frac{D\mathbf{u}_\alpha}{Dt} + \nabla\eta + \mu^2 \left\{\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3(z)\right\} + \mathbf{R} = 0 \tag{2}$$

$$\mathbf{R} = \mathbf{R}_f - \mathbf{R}_b - \mathbf{R}_l \tag{3}$$

The wave breaking terms are expressed by $\mathbf{R}_b$, while $\mathbf{R}_f$ model the bottom friction and $\mathbf{R}_l$ the subgrid mixing terms. The term $\mathbf{V}_3(z)$ introduces dependence of the equations from z, closure of the system depends on choice of method for evaluating this term. We here compare results from three different closures:

1. Wei et al. (1995): Neglects $\mathbf{V}_3$.
2. Chen et al. (2003): Evaluates $\mathbf{V}_3$ at $z = z_\alpha$
3. Chen (2006): In this model the momentum equations are integrated over depth.

The models differ in terms of the advection properties for vertical vorticity. The eulerian description of the flow field in rip and longshore currents case has been discussed in Gobbi et al. (2006). The differences in the predicted vorticity transport and flow organization, suggested to investigate the results in terms of the mixing features.

Wave-averaged model

It is well known that wave-averaged circulation models rely on a decomposition of the velocity $\mathbf{u}(x, y, z, t) = (u, v)$ in components corresponding to different time scales, following the approach in (Putrevu and Svendsen, 1999). We report here only the momentum equation in non conservative form for sake of brevity:

$$\tilde{\mathbf{U}}_{,t} + (\tilde{\mathbf{U}} \cdot \nabla)\tilde{\mathbf{U}} =$$

$$= -g\nabla\bar{\eta} - \frac{1}{gD}\nabla \cdot (\mathbf{S} - \mathbf{T} + \mathbf{L}) - \frac{1}{\rho D}\tau_B + \frac{1}{\rho D}\tau_S \qquad (4)$$

where $\bar{\eta}$ is the mean surface elevation with and h_0 the still water depth with. The local total water depth h, hence is $D = h_0 + \bar{\eta}$. $\tilde{\mathbf{U}}$ is the depth-uniform current velocity vector, $\mathbf{S}$, $\mathbf{L}$ and $\mathbf{T}$ are, respectively, the radiation stress tensor, the dispersive mixing tensor and the turbulent stress tensor, while τ_B and τ_S are the bottom and surface shear stress vectors respectively. Since Boussinesq type models do not retain a description of the vertical structure of the mean current the terms describing the vertical structure of the current are neglected also in the wave averaged model, being the comparison between the two the objective of the paper. Hence the original equations solved with SHORECIRC, coincide with non-linear Shallow water equations (NSWE).

NUMERICAL EXPERIMENTS

The study carried out here involved numerical tests using both a real bathymetry and an idealized one. The SANDYDUCK experiment was selected as it is suitable for studying particles dispersion in a longshore current over a relatively simple bottom configuration. However, before considering the real bathymetry, an idealized one will be taken into account to reduce the complexity of the problem introduced by a real sea bottom. This bathymetry is generated considering a cross-shore section of the surveyed SANDYDUCK one (the model bathymetry obtained from the survey is shown in Fig. 1). Also wave conditions have been simplified with respect to the ones relative to the time interval between $4:00am$ and $7:00am$ on Oct, 2^{nd} 1997.

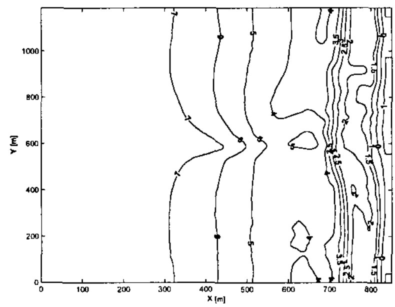

Figure 1. **Model bathymetry for SANDYDUCK**

The bathymetry is uniform along the y coordinate, repeating the cross-shore section located at $y = 267m$ in the model coordinate system shown in Fig. 1 (we here use model coordinates and not the FRF ones). The numerical mesh size is 567×298 with $\Delta x = 1.5m$ and $\Delta y = 4.0m$ and it is periodic in y. The wave attack has been assumed to be monochromatic with 1.06 m amplitude, and 6.6 s period and incidence angle at the generation point is $26°$. The turbulent mixing and bottom friction coefficients are 0.15 and 1.5×10^{-3}, respectively. All the simulations start from cold start and cover a 3 hours time interval. In order to study the mixing properties described by the models it is necessary to compute the trajectories of virtual particles displaced in the domain. For each model, the velocity field at each time step is stored and processed Particles trajectories are obtained by numerical integration of the Eulerian velocity components using a ODE solver (Runge Kutta forth order) implemented in MATLAB. The OdEs are simply: $Dx/dt = u$ and $Dy/dt = v$, where u and v are defined by (u, v)= $(u_\alpha$,$v_\alpha)$ for the BTMs and by considering both waves and currents in NSWE. Linear interpolation in space has been used. No interpolation in time is used, dt being the same as the one used for the numerical models time marching. The virtual particles are launched in randomly filled circular patches. In order to avoid the interaction with the lateral boundaries the cluster is released always at aa distance (usually $100m$) from one of the upstream lateral boundary. As in LaCasce and Bower (2000) the attention will be focused on the dispersion and diffusion of the particles. Once the trajectories $(x_i(x_0, t), x_j(x_0, t))$, $i, j = 1, 2$ (i,j=1, cross-shore direction; i,j=2, longshore direction) are known, it is possible to compute the elements of the absolute dispersion tensor:

$$\langle X_{ij}^2(t)\rangle = \frac{1}{M} \sum_{particles} x_i x_j \tag{5}$$

where x_i is the particle displacement from its initial position in the i^{th}-direction and in the j^{th}-direction and M is the number of particles in the cluster. The trace of this matrix is the total absolute dispersion, i.e. $\langle X_{tot}^2 \rangle = \langle X_{11}^2 \rangle + \langle X_{22}^2 \rangle$. The derivative in time of $\langle X_{tot}^2 \rangle$ is defined as the total absolute dispersion of the cluster$K$. If only the cross-shore and longshore elements are considered cross-shore and longshore diffusivity are obtained. In analogy with the absolute dispersion we define a relative dispersion. This is computed on pairs of particles located within a given distance at the time of the tracers release. The elements of the relative dispersion tensor are defined as:

$$\langle D_{ij}^2(t) \rangle = \frac{1}{N} \sum_{pairs} y_i y_j \tag{6}$$

where y_i is the separation between two particles in the same pair in the i^{th}-direction and in the j^{th}-direction and N is the number of pairs. As done for the absolute statistics, the trace of the tensor represents the total relative dispersion $\langle D_{tot}^2 \rangle = \langle D_{11}^2 \rangle + \langle D_{22}^2 \rangle$, hence a relative diffusion μ may be defined as the derivative of D_{tot}. The behavior of the relative dispersion and diffusivity in time and the relationship between these properties and particles/drifters separation (S), is often used as an indicator of the mixing regime of the flow. Of particular interest is the regime reached and before the relative and absolute diffusion both reach a constant value (browinian regime). In the presence of a background shear the turbulent relative diffusivity follows a $4/3$-law in this intermediate regime. Similar behaviors can also occur in the presence of waves. Johnson and Pattiaratchi (2004) found evidence of this law inside and outside the surf zone studying pulsating rip currents.

Longshore uniform barred beach

We here discuss the numerical experiments carried out using the uniform barred beach. In this case a cluster of 80 particles has been released on the bar, inside the surf zone in all the models considered here. In all the models the cluster is released after 2 hours and 15 minutes from the beginning of the simulation. Fig. 3 shows the trajectories of the particles as computed starting from the models results. Even though the trajectories displayed here come from a particular launch, other attempts have been made, all resulting in similar statistics for each of the model, even though the details of the trajectories may differ.

The virtual particles have been released at the beginning of the third hour of each simulation. The Eulerian field at that time is different for each model, details may be found in Gobbi et al. (2006) and will not been shown here. Inspection of the figure reveals several important differences.Results coming from Wei et al. (1995) show that all the particles are transported

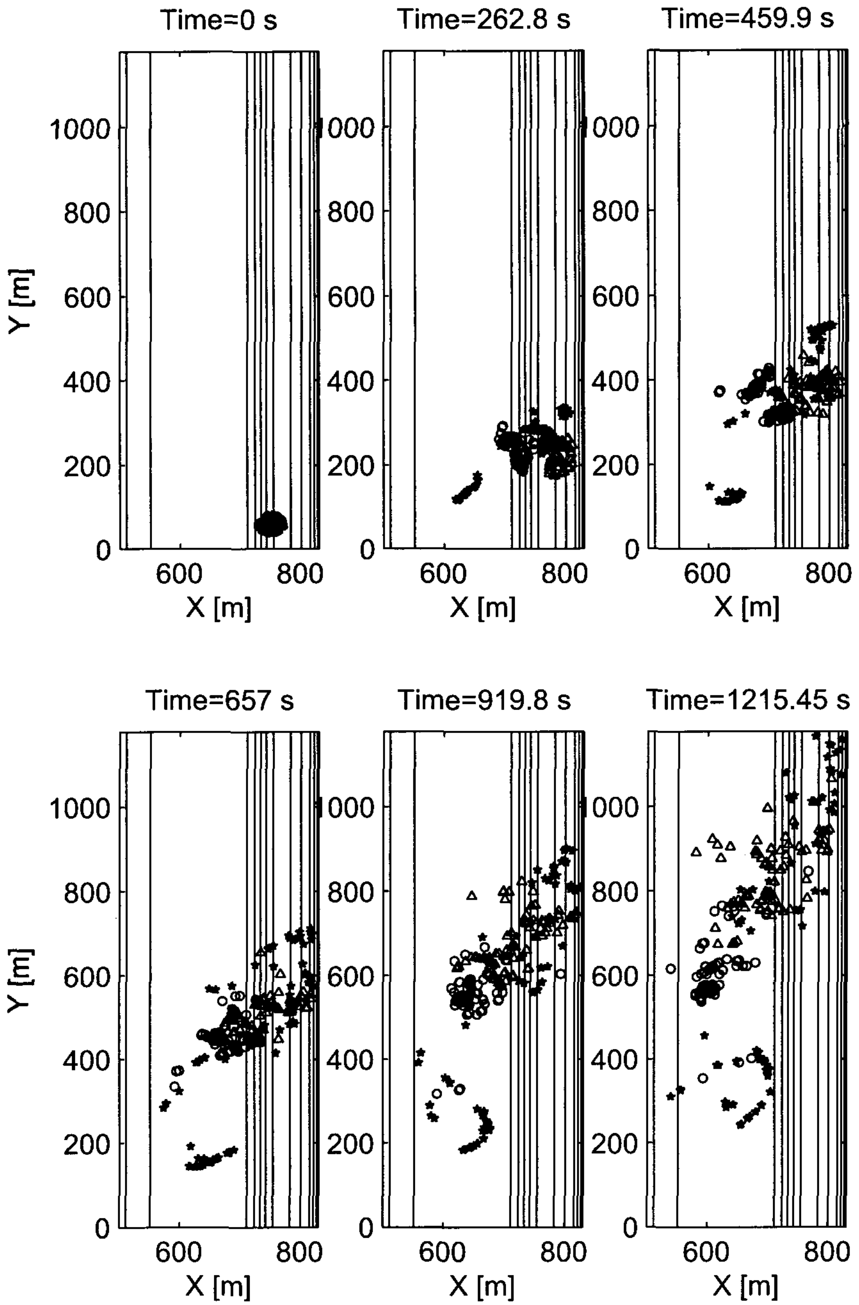

Figure 2. Particles trajectories on a longshore uniform barred breach. Circles Wei et al. (1995), triangles Chen et al. (2003), stars Chen (2006).

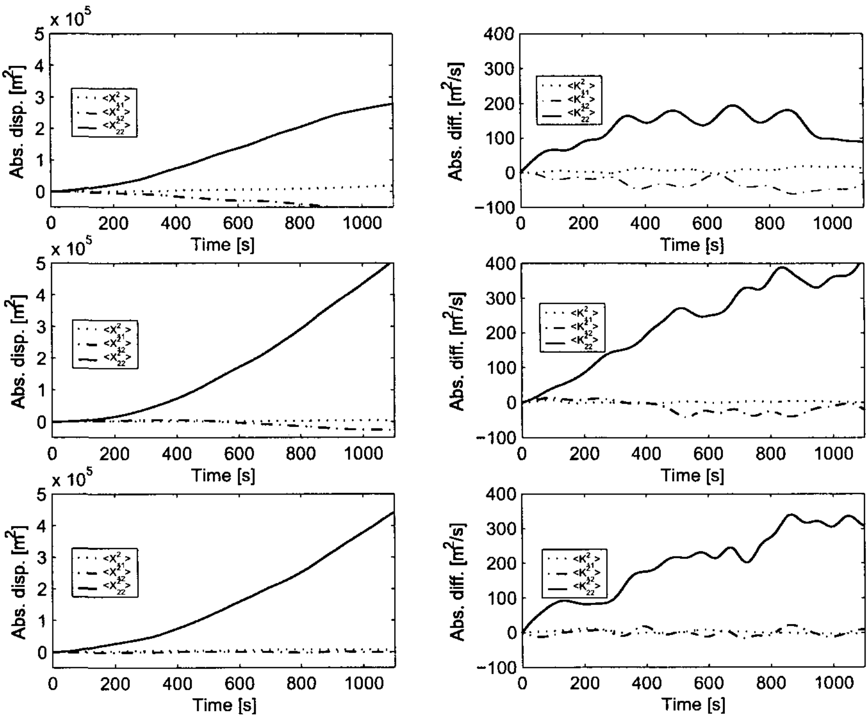

Figure 3. Absolute dispersion (left) and diffusion (right), longshore uniform barred breach. Upper panels Wei et al. (1995), central panels Chen et al. (2003), lower panels Chen (2006).

outside the surf zone with a smaller relative dispersion if compared to the other models. Instead the particles used in Chen et al. (2003) drift alongshore on the bar across the surfzone. Part of the cluster is transported in the through of the bar, while the part is drifted offshore by the current instabilities. Furthermore, Chen (2006) produces a similar scenario with respect to Chen et al. (2003), while part of the cluster is transported offshore by jets and part in the trough of the bar, the alongshore spreading of the particles is larger and the transport to the shoreline is stronger.

As it is evident, after about $1200s$ a large number of particles reaches the lateral boundary, hence, in order to avoid the influence of this region on the boundary, the results will include only the first $1100s$ of the simulations. Fig. 3 shows the elements of the absolute dispersion tensor ($\langle X^2_{ij} \rangle$) and absolute diffusivity ($\langle K_{ij} \rangle$) Absolute statistics (see Fig. 3) allow some consideration:

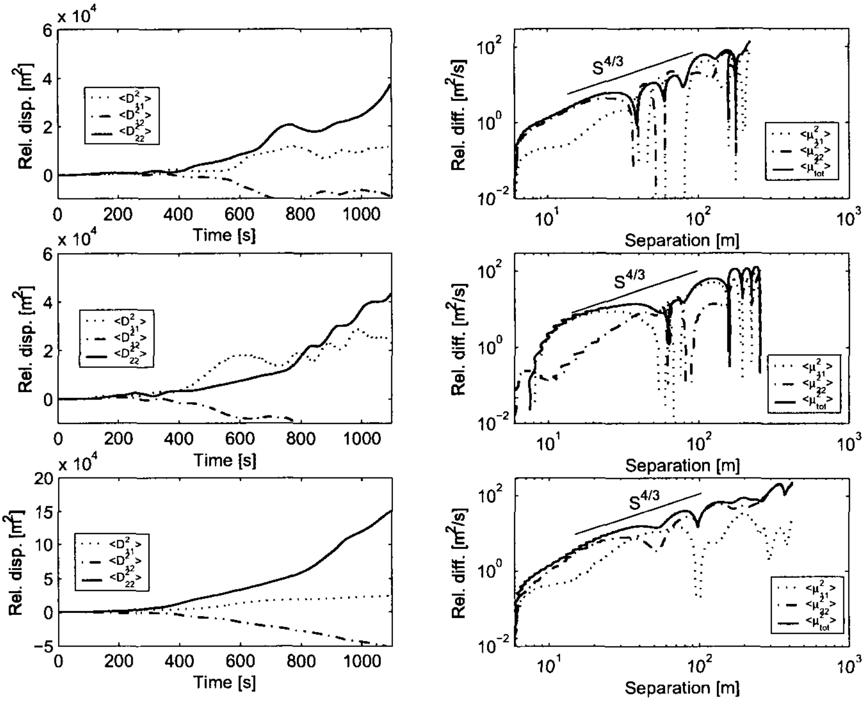

Figure 4. Relative dispersion (left) and diffusion (right), longshore uniform barred breach. Upper panels Wei et al. (1995), central panels Chen et al. (2003), lower panels Chen (2006).

1. Wei et al. (1995) model shows the lower longshore absolute dispersion, and the larger cross-shore one.
2. The cross-shore dispersion is from one to two orders of magnitude lower than the longshore one for each model.
3. Within the time interval in which the trajectories are studied, a constant value of longshore diffusivity seems to be reached for Chen et al. (2003) and Chen (2006) after $800s$, even though oscillations persist. For Wei et al. (1995) diffusivity oscillates around the value of $150m^2/s$ after $300s$ and decreases after $1000s$. However the existence of a plateau for the diffusivity is not clear in the interval studied as it is influenced by the mean longshore current which causes a growth in diffusivity.

More important indications are given by the relative (two particles) statistics shown in Fig.3. The relative statistics shown here are computed considering an initial separation smaller than $4m$. The results suggest

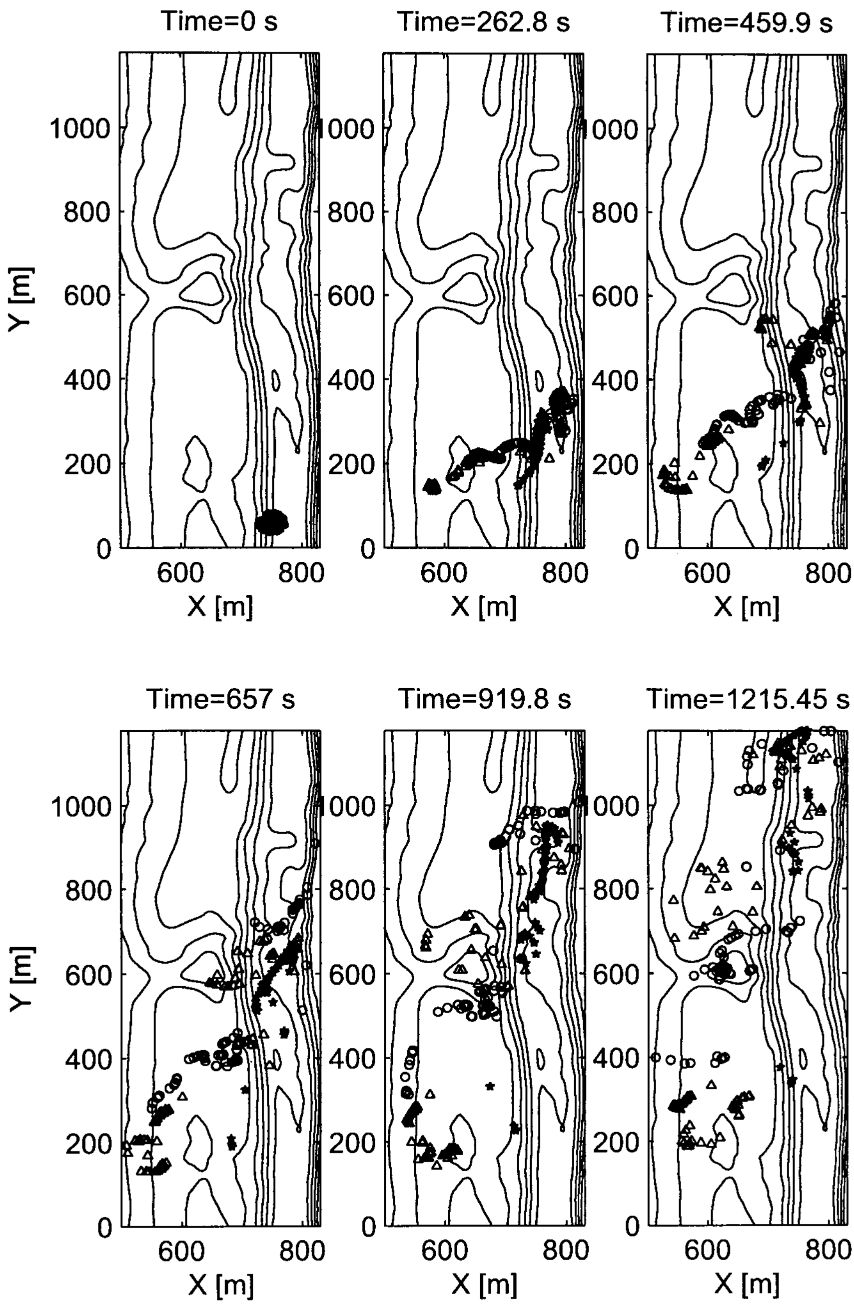

Figure 5. Particles trajectories on SANDYDUCK bathymetry. Circles Chen et al. (2003), triangles Chen (2006), stars NSWE.

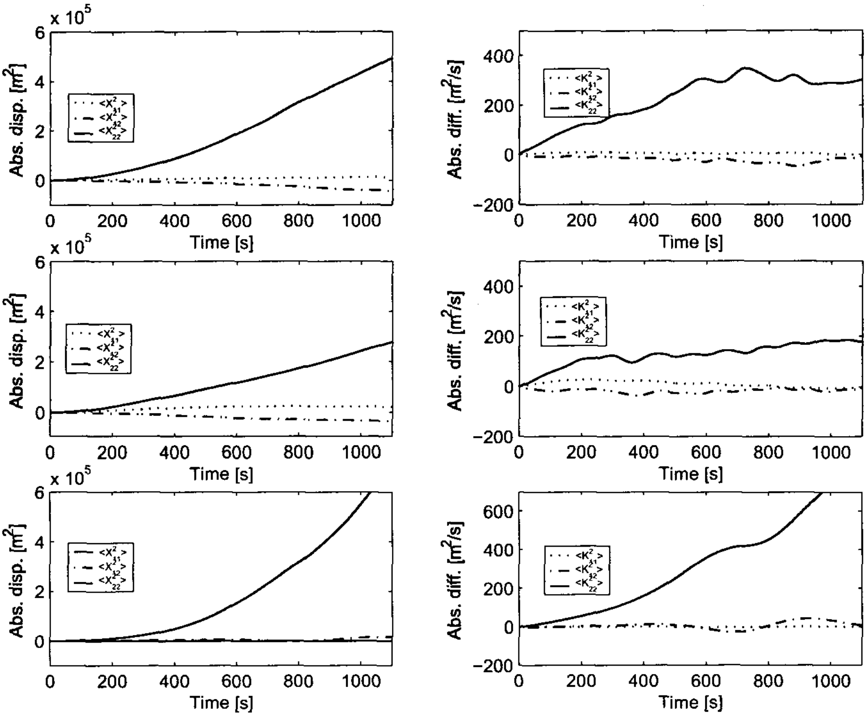

Figure 6. **Absolute dispersion for Sandyduck case. Upper panels Chen et al. (2003), central panels Chen (2006), lower panels NSWE.**

the existence of an intermediate regime close to a Richardson like $4/3$ law, for all the models, even though some results seem to support a weaker dependency. At separation smaller than $100m$, Chen et al. (2003) and Chen (2006) show a different relative importance of cross-shore and longshore dispersion/diffusivity. Longshore relative diffusion and dispersion dominates in Chen (2006), the opposite in Chen et al. (2003). Though this similarity exists from a statistical point of view between Wei et al. (1995) and Chen (2006) at small scales, it has to be reminded that the scenario described by the particles trajectories is totally different between the two models. Chen et al. (2003) and Chen (2006) trajectories describe a similar scenario, however, a large difference between the two models exists in term of relative statistics and in the magnitude of the longshore dispersion which is larger in Chen (2006). This may be explained by considering that the absolute statistics is influenced by the presence of particles that drift offshore in a zone where the longshore current is not effective. Besides, in Chen (2006) for separations larger than

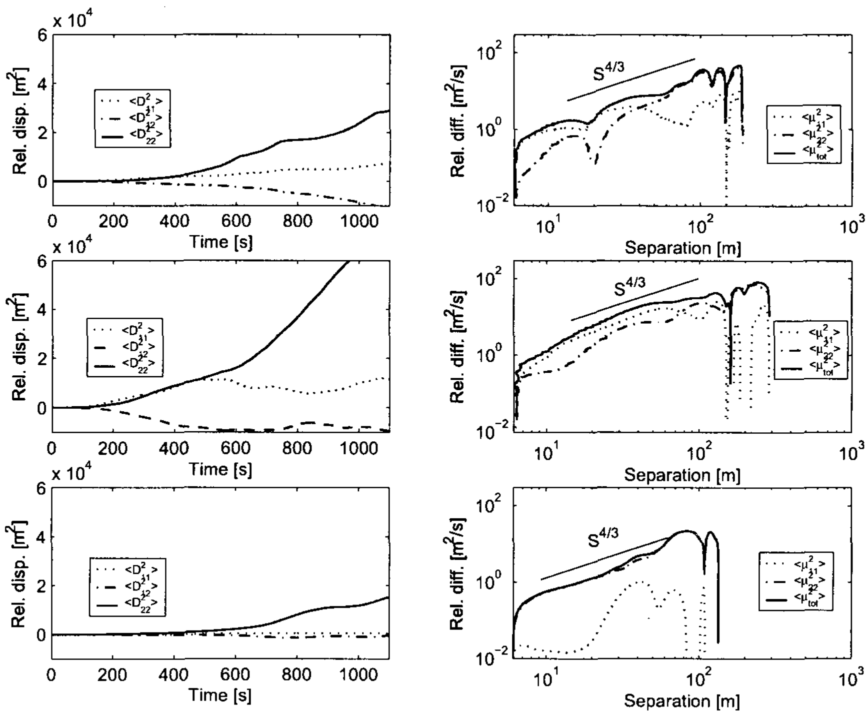

Figure 7. Relative dispersion for Sandyduck case. Upper panels Chen et al. (2003), central panels Chen (2006), lower panels NSWE.

about $100m$ a plateau characterizes cross-shore diffusivity while longshore one still grows. The figure also shows a comparison with a $4/3$ power law, which is well followed by Wei et al. (1995) and (Chen et al., 2003)

The Sandyduck case

In this section we discuss the simulation of the the propagation of regular waves over a real bathymetry coming from the SANDYDUCK experiments. The bathymetry is illustrated in Fig. 1. In this case only two BTEs were tested and also the results coming from the NSWE approach are shown. Fig. 3 shows the different trajectories coming from Chen et al. (2003), Chen (2006) and the NSWE. Most of the features found in the previous tests are confirmed here, the transport offshore is generally larger considering the real bathymetry and field wave conditions and in both BTMs is not localized in the rip channel only. The main differences arise from a weaker longshore current offshore the bar in Chen (2006). This cause the particles to drift offshore without a large longshore transport as in Chen et al. (2003). Totally different from these results are those coming from the NSWE. In this case, due to weaker instabilities, no vortex detachment from the longshore flow is detected and the transport is almost exclusively directed alongshore. As a result both the absolute dispersion (Fig.3) and the relative one (Fig.3) are dominated by the longshore component. Interesting results are given by the relative statistics. The total diffusivity in Chen (2006) seems to follow a power law with an exponent lower than $4/3$ for separations larger than $30 - 40m$ because of the contribution of the cross-shore diffusivity. The longshore diffusivity follows the aforementioned law. Also in Chen et al. (2003)the cross-shore diffusivity is important at separations up to $30 - 40m$, where the longshore diffusivity does not follow a $4/3$ law. The longshore diffusivity seems to reach a plateau for scales larger than $100m$. The $4/3$ law is well followed by the particles coming from the NSWE model.

CONCLUSIONS

The study has analyzed the trajectories of virtual particles released in the surf zone in numerically simulated shear waves. Three wave resolving Boussinesq type models with different vorticity transport properties have been compared using an idealized bathymetry and a real bathymetry coming from the SANDYDUCK field experiment. In this latter case, together with two among the selected BTMs, a wave averaged model based on NSWE (i.e. SHORECIRC without the vertical dispersion description) has been used. Results coming from a representative launch have been shown. The analysis of the trajectories and the Lagrangian statistics pointed out several differences among the models. Wei et al. (1995) predicts a very low transport inshore. On the other hand Chen et al. (2003) and Chen (2006) trajectories are quite close with each other. The main difference between the two models is the role played by the cross-shore transport directed offshore that is relatively more important in Chen (2006) case for the considered launch at small separations. The Richardson $4/3$ law is found reasonably well supported by the longshore relative diffusivity in the BTMs under study and in the wave averaged model even though some results suggest a lower exponent at large ($> 100m$) separations expecially for Chen (2006). This occurs in both the considered bathymetries. Results obtained with the wave-averaged model can only give few indications, the flow dynamics at time scales smaller than that of the wave being ignored by definition. However, given the weaker longshore current instabilities generated the cross-shore transport is very weak and the longshore dispersion dominates the process. Analysis is underway in order to overcome the limitation of the present version of the ODE solver at the lateral boundary and to explain the role of the shoreline boundary conditions in influencing the shear dispersion that is observed close to shoreline.

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