Landslide Benchmark Results for Non-Hydrostatic Wave Model NHWAVE

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Abstract

We provide a brief overview of the NHWAVE model and it's application to the study of seven benchmark problems established for the NTHMP Landslide Tsunami Model Benchmarking Workshop, held in January 2017 in Galveston, Texas. Results for benchmark tests 2, 4 and 7 are included in the main report that this is appended to. A full description of the model application to all seven benchmarks may be found in Zhang et al. (2017).

1 Introduction

NHWAVE is a non-hydrostatic wave-resolving model initially developed by Ma et al. (2012) to simulate the propagation of fully dispersive, fully nonlinear surface waves and resulting circulation in complex 3D coastal environments. The model is formulated in time-dependent, surface and terrain-following σ-coordinates, and can provide instantaneous descriptions of surface displacement and the three dimensional velocity and pressure fields. Wave breaking is handled naturally by the shock-capturing properties of the model's finite volume TVD formulation. The model is similar in intent to other non-hydrostatic solvers such as SWASH (Zijlema et al., 2011), and has been found to be of comparable or better accuracy than most similar models, particularly in terms of the number of vertical levels needed in application. NHWAVE has been extended to incorporate various landslide models in order to simulate tsunami wave generation by solid slides (Ma et al., 2012), multiphase simulation of suspended sediment load (Ma et al., 2013), granular debris flows (Ma et al., 2015) and viscous fluid slides (Kirby et al., 2016). This report provides an overview of the model equations and numerical approach. Landslide benchmark test results for benchmarks 1-7 (obtained from http://www1.udel.edu/kirby/landslide/problems.html) are described separately in Zhang et al. (2017). Results for benchmarks 2, 4 and 7 are included in the main report.

2 NHWAVE model equations

The governing equations of NHWAVE are the incompressible Navier-Stokes equations in well-balanced conservative form, formulated in time-dependent surface and terrain-following σ coordinate, which is defined as

$$t = t^* \quad x = x^* \quad y = y^* \quad \sigma = \frac{z^* + h}{D} \tag{1}$$

where the total water depth $D(x, y, t) = h(x, y, t) + \eta(x, y, t)$, $h(x, y, t)$ is the water depth with respect to the datum, which is temporally varying with landslides, $\eta(x, y, t)$ is the free surface elevation.

With σ coordinate transformation, the well-balanced mass and momentum equations are given by

$$\frac{\partial D}{\partial t} + \frac{\partial Du}{\partial x} + \frac{\partial Dv}{\partial y} + \frac{\partial \omega}{\partial \sigma} = 0 \tag{2}$$

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial \sigma} = \mathbf{S}_h + \mathbf{S}_p \tag{3}$$

where $\mathbf{U} = (Du, Dv, Dw)^T$ and ω is the vertical velocity in the σ coordinate image domain. The fluxes are given by

$$\mathbf{F} = \begin{pmatrix} Duu + \frac{1}{2}g\eta^2 + gh\eta \\ Duv \\ Duw \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} Duv \\ Dvv + \frac{1}{2}g\eta^2 + gh\eta \\ Dvw \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} u\omega \\ v\omega \\ w\omega \end{pmatrix}$$

The source terms on the right hand side of equation (3) account for the contributions from hydrostatic pressure and non-hydrostatic pressure respectively, written as

$$\mathbf{S}_h = \begin{pmatrix} g\eta\frac{\partial h}{\partial x} \\ g\eta\frac{\partial h}{\partial y} \\ 0 \end{pmatrix} \quad \mathbf{S}_p = \begin{pmatrix} -\frac{D}{\rho}(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial \sigma}\frac{\partial \sigma}{\partial x^*}) \\ -\frac{D}{\rho}(\frac{\partial p}{\partial y} + \frac{\partial p}{\partial \sigma}\frac{\partial \sigma}{\partial y^*}) \\ -\frac{1}{\rho}\frac{\partial p}{\partial \sigma} \end{pmatrix}$$

where p is the dynamic pressure.

To solve the water depth D, we integrate the continuity equation (2) from $\sigma = 0$ to 1. By using the boundary conditions at the bottom and surface for ω, we may obtain the equation for free surface movement.

$$\frac{\partial D}{\partial t} + \frac{\partial}{\partial x}(D\int_0^1 u d\sigma) + \frac{\partial}{\partial y}(D\int_0^1 v d\sigma) = 0 \tag{4}$$

3 Simulating slides as suspended sediment load

In this approach, landslides are simulated as water-sediment mixture, which can be diffused and diluted during their movement. The dense plume is driven by the baroclinic pressure forcing, which is introduced by the spatial density variation. The suspended sediment concentration can be computed from the convection-diffusion equation for suspended sediment load, which is given as follows in σ coordinate.

$$\begin{aligned}\frac{\partial DC}{\partial t} + \frac{\partial DuC}{\partial x} + \frac{\partial DvC}{\partial y} + \frac{\partial(\omega - w_s)C}{\partial \sigma} &= \frac{\partial}{\partial x}[D(\nu + \frac{\nu_t}{\sigma_h})\frac{\partial C}{\partial x}] \\ &+ \frac{\partial}{\partial y}[D(\nu + \frac{\nu_t}{\sigma_h})\frac{\partial C}{\partial y}] + \frac{1}{D}\frac{\partial}{\partial \sigma}[(\nu + \frac{\nu_t}{\sigma_v})\frac{\partial C}{\partial \sigma}]\end{aligned} \tag{5}$$

where C is the concentration of suspended sediment and w_s is sediment settling velocity. In the following, we will vary the sediment settling velocity to study its effects on landslide motion and associated tsunami waves. σ_h and σ_v are horizontal and vertical Schmidt numbers for sediment, respectively.

To solve the above equation, boundary conditions are needed to be specified at all the physical boundaries. Specifically, at the free surface, the vertical sediment flux is zero. At the bed-fluid interface, there is mass exchange of suspended sediment, which accounts for sediment erosion and deposition. However, in the following studies of submarine landslide, we assume that the submarine landslide is a self-sustained system. Thus, no mass exchange occurs at the bed. Therefore, a zero vertical flux boundary condition is imposed at both free surface and bottom.

$$(\nu + \frac{\nu_t}{\sigma_v})\frac{1}{D}\frac{\partial C}{\partial \sigma} + w_s C = 0 \tag{6}$$

4 Simulating slides as a discrete lower layer: Viscous slide equations

In this approach, landslides are simulated using a water layer (consisting of the basic NHWAVE model) overlying a depth-integrated slide layer). The slide layer here consists of a single depth-integrated model based on the equations for a highly viscous Neutonian fluid, following the work of Fine et al. (1998), and using a standard horizontal Cartesian coordinate system (x, y) referenced to the level still water surface and coinciding with the coordinate system used in NHWAVE. The sketch of this two-way coupled model is presented in Figure 1. The upper layer is freshwater with density ρ_f, surface elevation $\eta(x, y, t)$ and water depth h from still water to slide surface, while the lower layer is sediments with density ρ_s, dynamic viscosity μ_s, velocity $(u(x, y, t), v(x, y, t), w(x, y, t))$ and thickness of layer $D(x, y, t)$. Since the slide is bounded by its upper surface $z = -h(x, y, t)$ and the seabed surface $z = -h_s(x, y, t)$, the slide thickness $D(x, y, t) = h_s(x, y, t) - h(x, y, t)$. The long-wave regime, viscous regime and mild-slope assumption give the horizontal velocity profile

$$u(x, y, z, t) = \frac{3}{2} U(x, y, t)(2\xi - \xi^2) \tag{7}$$

$$v(x, y, z, t) = \frac{3}{2} V(x, y, t)(2\xi - \xi^2) \tag{8}$$

where U and V are depth-averaged horizontal velocity, and $\xi = \frac{z+h_s}{D}$ and represents seabed when $\xi = 0$ and slide surface when $\xi = 1$.

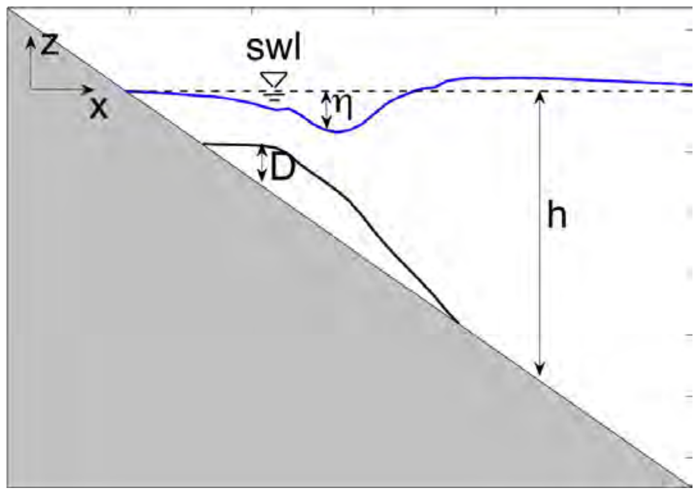

Figure 1: Definition sketch for underwater landslide. (From Grilli et al., 2016)

The resulting depth-averaged governing equations for the slide are given in conservative form by

$$\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S}_p + \mathbf{S}_h + \mathbf{S}_\tau + \mathbf{S}_f \tag{9}$$

where $\mathbf{E} = (D, DU, DV)^T$, and the fluxes are

$$\mathbf{F} = \begin{pmatrix} DU \\ \frac{6}{5} DUU + \frac{1}{2} g D^2 \\ \frac{6}{5} DUV \end{pmatrix} \qquad \mathbf{G} = \begin{pmatrix} DV \\ \frac{6}{5} DVU \\ \frac{6}{5} DVV + \frac{1}{2} g D^2 \end{pmatrix}$$

The source terms are given by

$$\mathbf{S}_h = \begin{pmatrix} 0 \\ -\frac{D}{\rho_s} \frac{\partial p_f}{\partial x} \\ -\frac{D}{\rho_s} \frac{\partial p_f}{\partial y} \end{pmatrix} \quad \mathbf{S}_p = \begin{pmatrix} 0 \\ gD \frac{\partial h_s}{\partial x} \\ gD \frac{\partial h_s}{\partial y} \end{pmatrix}$$

$$\mathbf{S}_\tau = \begin{pmatrix} 0 \\ \nu_s \left[D \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - 3\frac{U}{D} \right] \\ \nu_s \left[D \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) - 3\frac{V}{D} \right] \end{pmatrix} \quad \mathbf{S}_f = \begin{pmatrix} 0 \\ \frac{gn^2}{D^{1/3}} \sqrt{U^2 + V^2} U \\ \frac{gn^2}{D^{1/3}} \sqrt{U^2 + V^2} V \end{pmatrix}$$

where $\nu_s = \mu_s / \rho_s$ is kinematic viscosity of slide and n is Manning friction coefficient.

5 Simulating slides as a discrete lower layer: Granular slide equations

A version of the depth-integrated lower slide layer has been developed by Ma et al. (2015), based on the granular flow model of Iverson and Denlinger (2001). This version of the model has been developed in slope-oriented coordinates and is formulated only for a planar slope. Use of the model thus requires a mapping from slope-oriented coordinates to horizontal coordinates in order to locate slide properties properly in the NHWAVE grid. The governing equations for the granular slide are given by

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x'} + \frac{\partial \mathbf{G}}{\partial y'} = \mathbf{S} \tag{10}$$

in which (x', y') are slope-oriented coordinates, and $\mathbf{U} = (h_a, h_a u_a, h_a v_a)^T$. The fluxes are given by

$$\mathbf{F} = \begin{pmatrix} h_a u_a \\ h_a u_a^2 + \frac{1}{2}[(1-\lambda)k_{act/pass} + \lambda] g_{z'} h_a^2 \\ h_a u_a v_a \end{pmatrix} \tag{11}$$

$$\mathbf{G} = \begin{pmatrix} h_a v_a \\ h_a u_a v_a \\ h_a v_a^2 + \frac{1}{2}[(1-\lambda)k_{act/pass} + \lambda] g_{z'} h_a^2 \end{pmatrix} \tag{12}$$

The source term is

$$\mathbf{S} = \begin{pmatrix} 0 \\ S_{x'} \\ S_{y'} \end{pmatrix} \tag{13}$$

where

$$\begin{aligned} S_{x'} =& g_{x'} h_a - \frac{h_a}{\rho} \frac{\partial P_h^f}{\partial x'} - (1-\lambda) g_{z'} h_a \tan \phi_{bed} \frac{u_a}{\sqrt{u_a^2 + v_a^2}} \\ & - sgn\left(S_{x'y'}\right) h_a \frac{\partial}{\partial y'} [g_{z'} h_a (1-\lambda)] \sin \phi_{int} \end{aligned} \tag{14}$$

$$\begin{aligned} S_{y'} =& g_{y'} h_a - \frac{h_a}{\rho} \frac{\partial P_h^f}{\partial y} - (1-\lambda) g_{z'} h_a \tan \phi_{bed} \frac{v_a}{\sqrt{u_a^2 + v_a^2}} \\ & - sgn\left(S_{y'x'}\right) h_a \frac{\partial}{\partial x'} [g_{z'} h_a (1-\lambda)] \sin \phi_{int} \end{aligned} \tag{15}$$

where $g_{z'}$ is the component of gravitational acceleration normal to the slope, $k_{act/pass}$ is the Earth pressure coefficient given by Ma et al. (2015), ϕ_{bed} is the friction angle of the granular material contacting the bed, ϕ_{int} is the internal friction angle of the granular solid, λ is a parameter to be determined.

6 Numerical method

The well-balanced continuity and momentum equations (2) and (3) are discretized by a combined finite-volume and finite-difference approach with a second-order Godunov-type scheme. Following the numerical framework of NHWAVE (Ma et al., 2012), the velocities are defined at the cell centers, while the pressure is defined at the vertically-facing cell faces in order to accurately prescribe zero pressure condition at the free surface. The two-stage second-order nonlinear Strong Stability-Preserving (SSP) Runge-Kutta scheme (Gottlieb et al., 2001) is adopted for time stepping in order to obtain second-order temporal accuracy. At the first stage, an intermediate quantity $\mathbf{U}^{(1)}$ is evaluated using a typical first-order, two-step projection method given by

$$\frac{\mathbf{U}^{*}-\mathbf{U}^{n}}{\Delta t} = -\left(\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}+\frac{\partial \mathbf{H}}{\partial \sigma}\right)^{n}+\mathbf{S}_{h}^{n} \tag{16}$$

$$\frac{\mathbf{U}^{(1)}-\mathbf{U}^{*}}{\Delta t} = \mathbf{S}_{p}^{(1)} \tag{17}$$

where $\mathbf{U}^{n}$ represents $\mathbf{U}$ value at time level n, $\mathbf{U}^{*}$ is the intermediate value in the two-step projection method, and $\mathbf{U}^{(1)}$ is the final first stage estimate.

At the second stage, the velocity field is updated to a second intermediate level using the same projection method, after which the Runge-Kutta algorithm is used to obtain a final value of the solution at the $n+1$ time level.

$$\frac{\mathbf{U}^{*}-\mathbf{U}^{(1)}}{\Delta t} = -\left(\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}+\frac{\partial \mathbf{H}}{\partial \sigma}\right)^{(1)}+\mathbf{S}_{h}^{(1)} \tag{18}$$

$$\frac{\mathbf{U}^{(2)}-\mathbf{U}^{*}}{\Delta t} = \mathbf{S}_{p}^{(2)} \tag{19}$$

$$\mathbf{U}^{n+1} = \frac{1}{2}\mathbf{U}^{n}+\frac{1}{2}\mathbf{U}^{(2)} \tag{20}$$

In the projection step (16) - (17) and (14)), a Poisson equation can be derived by applying the continuity equation (Ma et al., 2012). The Poisson equation is discretized by finite difference method, resulting in a linear equation system with a coefficient matrix of 15 diagonal lines. The linear system is solved using the high performance preconditioner HYPRE software library.

References

Fine, I., Rabinovich A., Kulikov E., Thomson R. and Bornhold B., 1998, “Numerical modelling of landslide-generated tsunamis with application to the Skagway Harbor tsunami of November

3, 1994", *Proc.International Conference on Tsunamis, Paris*, 211-223.

Gottlieb, S., Shu C.-W., and Tadmore, E., 2001, "Strong stability-preserving high-order time discretization methods", *SIAM Review*, **43** (1), 89 - 112.

Iverson, R. M. and Denlinger, R. P., 2001, "Flow of variably fluidized granular masses across three-dimensional terrain 1. Coulomb mixture theory", *Journal of Geophysical Research*, **106**, 537-552

Kirby, J. T., Shi, F., Nicolsky, D. and Misra, S., 2016, "The 27 April 1975 Kitimat, British Columbia submarine landslide tsunami: A comparison of modeling approaches", *Landslides*, **13**, 1421-1434, doi:10.1007/s10346-016-0682-x.

Ma, G., Shi F. and Kirby J.T., 2012, "Shock-capturing non-hydrostatic model for fully dispersive surface wave processes", *Ocean Modelling*, **43**, 22-35.

Ma, G., Kirby, J. T. and Shi, F., 2013, "Numerical simulation of tsunami waves generated by deformable submarine landslides", *Ocean Modelling*, **69**, 146-165, doi:10.1016/ j.ocemod.2013.07.001.

Ma G., Kirby J.T., Hsu T.-J. and Shi F., 2015, "A two-layer granular landslide model for tsunami wave generation: Theory and computation", *Ocean Modelling*, **93**, 40-55, doi:10.1016/ j.ocemod.2015.07.012.

Zhang, C., Kirby, J. T., Ma, G., Shi, F., Grilli, S. T. and Shelby, M., 2017, "NTHMP landslide benchmark results for NHWAVE, Version 3.0", Research Report No. CACR-17-05, Center for Applied Coastal Research, Department of Civil and Environmental Engineering, University of Delaware.