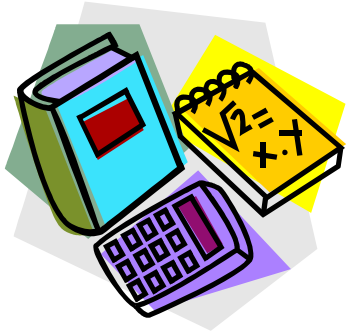
Math
Grade-Level Expectations



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Mathematics Standards

Acknowledgements

Thank you to everyone who participated in the development of the Mathematics Grade Level Expectations. Many wrote and edited particular grade level expectations, some participated in the review process, and some provided helpful comments about wording, glossary items, and grade appropriate information. All of your help was invaluable. The result is a much-appreciated (and long awaited) product that will provide Delaware teachers with a worthwhile curriculum alignment.

Additional thanks go to Deb Roberson and Jackie Edge for their clerical and technological assistance. Thanks to Amelia Hodges and Mike Stetter for their professional advice and support.

Preface

The original Mathematics Content Frameworks were adopted in 1995 and included Performance Indicators that described what students should know and be able to do by the end of a cluster of grades: K-3, 4-5, 6-8, and 9-10. In 2005, the Grade Level Expectations (GLEs) were developed to provide clarity for classroom teachers as to their specific responsibilities in guiding students to meet and exceed the Mathematics Content Standards. These Grade Level Expectations will be used at the local level to develop curricula and assessments to monitor progress towards meeting and exceeding the standards. At the state level they will be used to develop or identify items for inclusion on the Delaware Student Testing Program (DSTP). Standards were reorganized, but the language of the standards remained the same.

The standards from 1995 have been reorganized, but the language of the standards has remained the same. Computation has now been combined with Number Sense to form Numeric Reasoning. Algebra and Patterns, Relations, and Functions have been combined to form Algebraic Reasoning. Measurement has been pulled out of the Standard with Computation and combined with Spatial Sense and Geometry to form Geometric Reasoning. Statistics and Probability has been renamed Quantitative Reasoning.

Mathematics Standards

Content standards:

Standard 1 – Numeric Reasoning: Students will develop **Numeric Reasoning** and an understanding of *Number and Operations* by solving problems in which there is a need to represent and **model real number**s verbally, physically, and symbolically; to **explain** the relationship between numbers; to determine the relative magnitude of **real number**s; to use operations with understanding; and to select appropriate methods of calculations from among mental math, paper-and-pencil, calculators, or computers.

Standard 2 – Algebraic Reasoning: Students will develop **Algebraic Reasoning** and an understanding of Patterns and Functions by solving problems in which there is a need to recognize and extend a variety of patterns; to progress from the concrete to the abstract using physical **model**s, equations, and graphs; to describe, represent, and analyze relationships among variable quantities; and to analyze, represent, **model**, and describe real-world **functional relationship**s.

Standard 3 – Geometric Reasoning: Students will develop **Geometric Reasoning** and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.

Standard 4 – Quantitative Reasoning: Students will develop **Quantitative Reasoning** and an understanding of Data Analysis and Probability by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to **model** mathematical situations to determine the probability.

Process Standards:

Standard 5 – Problem Solving: Students will develop their Problem Solving ability by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply **strategies** to **solve** a wide variety of problems; and to integrate mathematical reasoning, communication and connections.

Standard 6 – Reasoning and Proof: Students will develop their Reasoning and Proof ability by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to **justify** their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking; and to construct their own learning.

Standard 7 – Communication: Students will develop their mathematical Communication ability by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats.

Standard 8 – Connections: Students will develop mathematical Connections by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways.

Mathematics Design Team – *Understanding by Design* (UbD)
Enduring Understandings and Essential Questions

Enduring Understandings:

Numeric Reasoning

1. Numbers can be represented in multiple ways.
2. The same operations can be applied in problem situations that seem quite different from one another.
3. Being able to compute fluently means making smart choices about which tools to use and when to use them.
4. Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.

Algebraic Reasoning

1. Change is fundamental to understanding functions.
2. Numbers or objects that repeat in predictable ways can be described or generalized.
3. An operation can be "undone" by its inverse.
4. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

Geometric Reasoning

1. Two- and three-dimensional objects can be described, classified, and analyzed by their attributes.
2. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape.
3. An object's location on a plane or in space can be described quantitatively.
4. Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other.

Quantitative Reasoning

1. The question to be answered determines the data to be collected and how best to collect it.
2. Basic statistical techniques can be used to analyze data in the workplace.
3. The probability of an event can be used to predict the probability of future events.

Process Standards

1. Mathematics can be used to solve problems outside of the mathematics classroom.
2. Mathematics is built on reason and always makes sense.
3. Reasoning allows us to *make* conjectures and to *prove* conjectures.
4. Classifying helps us build networks of mathematical ideas.
5. Precise language helps us express mathematical ideas and receive them.

Essential Questions:

Numeric Reasoning

1. What makes an estimate *reasonable*?
2. What makes an answer *exact*?
3. What makes a strategy both *effective* and *efficient*?
4. What makes a solution *optimal*?

Algebraic Reasoning

1. How can change be described mathematically?
2. How are patterns of change related to the behavior of functions?
3. How do mathematical models/representations shape our understanding of mathematics?

Geometric Reasoning

1. How are measurement and counting related?
2. How does *what* we measure affect *how* we measure? How can space be defined through numbers/measurement?
3. Why do we compare contrast and classify objects?
4. How do decomposing and recomposing shapes help us build our understand of mathematics?
5. How can transformations be described mathematically?

Quantitative Reasoning

1. What is average?
2. What makes a data representation useful?
3. How does my sample affect confidence in my predication?
4. What is fair?

Process Standards

1. Is your plan working? Do you need to reconsider what you are doing?
2. How are *solving* and *proving* different? How are *showing* and *explaining* different?
3. How do you know when you have proven something?
4. What does it take to **verify** a conjecture? How do you develop a convincing argument?
5. How do you make sense of different strategies? How do you determine their strengths and weaknesses? How do you determine similarities and differences?
6. Why do we classify? Why do we classify numbers? Why do we classify geometric objects? Why do we classify functions?

Standard 1 (K–5) – Numeric Reasoning: Students will develop Numeric Reasoning and an understanding of *Number and Operations* by solving problems in which there is a need to represent and **model real number**s verbally, physically, and symbolically; to **explain** the relationship between numbers; to determine the relative magnitude of **real number**s; to use operations with understanding; and to select appropriate methods of calculations from among mental math, paper-and-pencil, calculators, or computers.
Enduring Understandings: Numbers can be represented in multiple ways. The same operations can be applied in problem situations that seem quite different from one another. Being able to compute fluently means making smart choices about which tools to use and when to use them. Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.
Essential Questions: What makes an estimate *reasonable*? What makes an answer *exact*? What makes a strategy both *effective* and *efficient*? What makes a solution *optimal*?

All students in K**indergarten** will be able to:	Building upon the kindergarten expectations, all students in **Grade 1** will be able to:	Building upon the K–1 expectations, all students in **Grade 2** will be able to:	Building upon the K–2 expectations, all students in **Grade 3** will be able to:	Building upon the K–3 expectations, all students in **Grade 4** will be able to:	Building upon the K–4 expectations, all students in **Grade 5** will be able to:
Number sense: • Count sets of objects up to 20 • Connect number words and numerals (up to 10) to the quantities they represent using various physical **model**s and representations • Sequence numbers and **explain** what comes before, after and between other numbers ▪ Show more than one way to make numbers up to 10 ***Operations:*** • Use **manipulatives** to **model** putting together and taking apart (e.g., you have one cookie and you get two more cookies • Use **manipulatives** to show more than one way to make a target number up to 6	***Number sense:*** • Count sets of objects up to 50 by 1s, 2s, 5s, and 10s • Connect number words and numbers (up to 50) to the quantities they represent using physical **model**s and representations • Sequence numbers and **explain** which is larger, which is smaller, and what is between other numbers up to 100 • Compose and **decompose** numbers up to 20 ***Operations:*** • Use **manipulatives** and pictures to **model** putting together and taking apart numbers up to 20 • Write number sentences to represent addition combinations up to 10 • Use **manipulatives** and **model**s to demonstrate doubles • Use direct **model**s, **manipulatives** and pictures to demonstrate	***Number sense:*** • Develop efficient **strategies** for counting (e.g., skip counting by 1s, 2s, 5s and 10s) • Demonstrate an understanding that our number system is based on combinations of ones and tens—place value • Use combinations of one- and two-digit numbers to build larger (two-digit) numbers • Use multiple **strategies** to compare size of two numbers (counting up, counting back) • Connect number words for fractions with pictures and numerals (1/2, 1/3, 1/4) ***Operations:*** • Use number sentences to represent number combinations up to 20 • Use number sentences with missing addends to represent number combinations up to 20 • Use a variety of **strategies** to **solve** combination and	***Number sense:*** • Demonstrate an understanding that our number system is based on combinations of 1s, 10s, and 100s—place value • Connect counting up and counting back to addition and subtraction • Connect skip counting to multiplication • Develop understanding of fractions as parts of unit wholes • Compare the size of common fractions using **model**s ***Operations:*** • Add and subtract numbers up to 100 efficiently and **explain** the **strategies** used • Master addition and subtraction facts up to 20 • Develop and use **strategies** to estimate the results of addition and subtraction operations on **whole numbers**	***Number sense:*** • **Decompose** and **recompose whole numbers** up to 10,000 using a variety of one, two- and three-digit combinations • Determine factor pairs that make up a given number • Develop an understanding of fractions as parts of unit wholes and division of **whole numbers** • Demonstrate equivalent forms of common fractions using physical models, pictures, and number lines • Compare and order fractions using physical models, pictures, and number lines • Use decimal notation to show the value of coins • Explore negative numbers by extending the number line using familiar applications (elevator, temperature, sea level, debt)	***Number sense:*** • Describe **whole numbers** up to 100,000 using place value structure • Develop understanding of fractions as parts of unit wholes, as part of a collection, as locations on number lines, and as division of **whole numbers** • Describe numbers according to characteristics such as evens, odds, factors, multiples, and squares • Find 1/10 or 10 times a number using mental math • Generate and connect equivalent forms of benchmark fractions, decimals and percents • Use multiple models and methods to compare decimals • Use decimal form to represent benchmark fractions (1/3s, 1/4s, 1/5s, 1/10s)

All students in K**indergarten** will be able to:	Building upon the kindergarten expectations, all students in **Grade 1** will be able to:	Building upon the K–1 expectations, all students in **Grade 2** will be able to:	Building upon the K–2 expectations, all students in **Grade 3** will be able to:	Building upon the K–3 expectations, all students in **Grade 4** will be able to:	Building upon the K–4 expectations, all students in **Grade 5** will be able to:
	joining and separating problems	separation problems up to 100 • Show number sentences that demonstrate that addition and subtraction are **inverse operations** (e.g., join, separate, part-part-whole, compare) • Represent repeated addition using pictures and **model**s • Understand that addition of **whole numbers** result in a larger number and subtraction of **whole numbers** result in a smaller number	• Use pictures and number sentences to represent multiplication and division problems • Develop the concept of multiplication by using **model**s to represent and count the number of groups and the number of elements in each group (e.g., repeated addition, arrays, skip counting) • Select and use appropriate methods and tools for computing (e.g., mental computation, estimation, calculators, paper and pencil) depending on the context and nature of the computation	***Operations:*** • Choose the appropriate operation to **solve** a word problem and **explain** why • Add and subtract larger numbers (e.g., three digits + two digits) and **explain** how the operation works • Demonstrate mastery of mental math **strategies** for multiplying numbers (e.g., 25 × 8) • Show how multiplication and division facts up to 50 are related, using arrays, skip counting, and area models • Master multiplication facts and the related division facts up to the 10s tables • **Explain** the meaning of the remainder in a division problem based on the context of the problem • Develop and use **strategies** to estimate the results of operations on **whole numbers** • Use physical models and pictures to add and subtract benchmark fractions • Find 1/3, 1/4, and 1/5 of a given set or area using models • Add and subtract decimals using money models	• Use multiple methods and models to convert decimals to fractions and fractions to decimals • Develop the meaning of percent as a ratio of a number out of 100 • Use a variety of familiar applications to represent positive and negative numbers as opposites ***Operations:*** • Apply more than one operation to **solve** a word problem • Multiply and divide by large numbers (e.g., two digits by two digits) and show why the operation works • Use multiplication clusters to build mental math **strategies** (e.g., 5×2, 5×20, 50×2, 50×20) • Use partial products to **verify** how multiplication **algorithm**s work • Use and apply various meanings of multiplication and division (e.g., fair share, repeated addition/ subtraction, compare, rate) • Develop and use **strategies** to estimate the results of operations on **whole numbers** • Add and subtract benchmark fractions and fractions with

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				• Select and use appropriate methods and tools for computing (e.g., mental computation, estimation, calculators, paper and pencil) depending on the context and nature of the computation	common denominators using physical models • Multiply fractions by **whole numbers** using models such as: clock fractions, number/ratio tables, number lines, fractions strips, skip counting or array models • Multiply numbers by 10, 1/10th, 100 and 1/100th using mental math • Connect multiplication by 1/3, 1/4, 1/5 to division by its inverse (3, 4, 5) (e.g., $12 \times 1/4 = 12 \div 4$) • Find benchmark percents of numbers using physical models • Add and subtract decimals using models • Add and subtract **integers** using familiar applications such as sea level, elevators, etc. • Select and use appropriate methods and tools for computing (e.g., mental computation, estimation, calculators, paper and pencil) depending on the context and nature of the computation

Standard 1 (6–11) – Numeric Reasoning: Students will develop Numeric Reasoning and an understanding of *Number and Operations* by solving problems in which there is a need to represent and model **real number**s verbally, physically, and symbolically; to **explain** the relationship between numbers; to determine the relative magnitude of **real number**s; to use operations with understanding; and to select appropriate methods of calculations from among mental math, paper-and-pencil, calculators, or computers.
Enduring Understandings: Numbers can be represented in multiple ways. The same operations can be applied in problem situations that seem quite different from one another. Being able to compute fluently means making smart choices about which tools to use and when to use them. Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.
Essential Questions: What makes an estimate *reasonable*? What makes an answer *exact*? What makes a strategy both *effective* and *efficient*? What makes a solution *optimal*?

Building upon the K–5 expectations, all students in **Grade 6** will be able to:	Building upon the K–6 expectations, all students in **Grade 7** will be able to:	Building upon the K–7 expectations, all students in **Grade 8** will be able to:	Building upon the K–8 expectations, all students in **Grade 9** will be able to:	Building upon the K–9 expectations, all students in **Grade 10** will be able to:	Building upon the K–10 expectations, all students in **Grade 11** will be able to:
Number sense: • Expand understanding of the number system to include numbers in the millions • Use factors and multiples to demonstrate part whole relationships • Use factors and multiples to develop equivalent fraction families • Scale up or scale down fraction and whole number measurements (e.g., recipes) • Use place value structure to describe the size of decimals • Demonstrate equivalence of decimals, fractions, and percents using multiple models ***Operations:*** • Multiply fractions by **whole numbers** and **explain** the result • Multiply fractions by other fractions using physical models, ratio/rate tables, and arrays	***Number sense:*** • Use scientific notation to represent large numbers and decimals • Use powers of ten to represent place value • Interpret and apply percents greater than 100 • Use proportional reasoning to express rates (e.g., speed, density, mpg) • Compare fractions, decimals, and percents using multiple models • Explore the effects of scaling up and scaling down on the coordinate plane • Compare **integers** on the number line • **Explain** the relationship of a number to its additive inverse • Apply knowledge of **integers** to the coordinate plane ***Operations:*** • Multiply and divide fractions and use models to **justify** your solution	Number sense: • Use exponential notation to represent **whole numbers** • Use square numbers and square roots to reason about the relationship between the side of a square and area of the square (e.g., side of a square with an area of 9 is $\sqrt{9}$ or 3, side of a square with area of 5 is $\sqrt{5}$) • Apply knowledge of factors and multiples, evens and odds, primes and composites, to generalizations • Explore the meaning of **ir**rational number**s** such as π, or $\sqrt{3}$ ***Operations:*** • Perform computations with exponents, powers of 10, and scientific notation • Use **inverse operations** to "do and undo" mathematical operations with **rational number**s • Demonstrate the reasonableness of an	Number sense: • Demonstrate an understanding of numbers as rational or irrational • Compare relative sizes of **real number**s • Estimate square roots • Interpret absolute value as distance from zero • Determine the appropriateness of an answer by using number sense or estimation ***Operations:*** • Represent and operate with very large and very small numbers • Make generalizations about the effect of operations on **rational number**s • Use properties of the **real number** system to simplify expressions (Associative, Commutative, Identity, Inverse, and Distributive) • Recognize and use **inverse operations** to **solve** equations (e.g. addition and subtraction, multiplication and	Number sense: • Compare and contrast the properties of numbers and number systems, including the rational and **real number** systems • Determine the effect of using exact values or estimates for repeating decimals and **ir**rational number**s** (e.g. 2/3 or .67, π or 3.14) in a problem solving situation • Use square numbers and square roots in problem solving situations • Simplify numeric and symbolic expressions involving absolute value, square roots, and exponents ***Operations:*** • Use technology to add, subtract, and multiply with **matrice**s in problem solving situations	Number sense: • Compare and contrast the properties of numbers and number systems, including the rational and **real number** systems • Use $i = \sqrt{-1}$ to extend the **real number** system to represent the complex number system • Use sigma notation • Simplify expressions with negative and fractional exponents ***Operations:*** • Make generalizations about the effect of operations on **real number**s • Perform addition, subtraction, multiplication, and division on polynomial expressions • Perform addition, subtraction, multiplication, and division on radical expressions • Perform addition, subtraction, multiplication, and

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• Connect multiplication by a unit fraction (such as 1/3,1/4, 1/5, 1/10, 1/100) to division by its multiplicative inverse (3, 4, 5, 10, 100) using models • Add and subtract fractions with unlike denominators and use physical models to **justify** your answer • Calculate the decimal equivalent of fractions • Use **benchmark** percents to **solve** problems • **Explain** the role of place value in adding and subtracting decimals • Multiply decimals to **solve** real-world problems (e.g., find the cost of 3 1/2 pounds of grapes at $1.95 per pound) • Describe in which situations an estimate is preferable and in which situations the exact answer is required • Select and use appropriate methods and tools for computing (e.g., mental computation, estimation, calculators, paper, and pencil) depending on the context and nature of the computation	• Use a variety of **strategies** to add, subtract, multiply, and divide fractions • Calculate unit rate to **solve** real-world problems (e.g., speed of a car, unit price of food, etc.) • **Justify** the placement of the decimal point in the solution to a multiplication or division problem • Use ratios, proportions and percents to **solve** contextualized problems • Connect addition and subtraction of **integers** using models or concrete examples • Add, subtract, multiply, and divide **integers** • Use an estimation or mental math strategy to demonstrate the reasonableness on an exact answer • Select and use appropriate methods and tools for computing (e.g., mental computation, estimation, calculators, paper and pencil) depending on the context and nature of the computation	exact calculation by using an estimation or mental math strategy • **Explain** how the distributive property is used to multiply (e.g., partial products, mixed numbers) • Use meaningful relationships between addition, subtraction, multiplication, and division of **integers** to **justify** the rules of operations • Apply proportional reasoning **strategies** to **solve** real-world problems • Select and use appropriate methods and tools for computing (e.g., mental computation, estimation, calculators, paper and pencil) depending on the context and nature of the computation	division, squares and square roots) • Select and use appropriate methods and tools for computing from among mental computation, estimation, calculators, paper and pencil, and computers according to the context and nature of the computation		division on irrational expressions • Simplify powers of/and expressions involving operations with complex numbers

Standard 2 (K–5) – Algebraic Reasoning: Students will develop Algebraic Reasoning and an understanding of Patterns and Functions by solving problems in which there is a need to recognize and extend a variety of patterns; to progress from the concrete to the abstract using physical **model**s, equations, and graphs; to describe, represent, and analyze relationships among variable quantities; and to analyze, represent, **model**, and describe real-world **functional relationship**s.
Enduring Understandings: Change is fundamental to understanding functions. Numbers or objects that repeat in predictable ways can be described or generalized. An operation can be "undone" by its inverse. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
Essential Questions: How can change be described mathematically? How are patterns of change related to the behavior of functions? How do mathematical models/representations shape our understanding of mathematics?

All students in K**indergarten** will be able to:	Building upon the kindergarten expectations, all students in **Grade 1** will be able to:	Building upon the K–1 expectations, all students in **Grade 2** will be able to:	Building upon the K–2 expectations, all students in **Grade 3** will be able to:	Building upon the K–3 expectations, all students in **Grade 4** will be able to:	Building upon the K–4 expectations, all students in **Grade 5** will be able to:
Patterns and change: • Sort objects by a given **attribute** (e.g., size, color, shape) • Repeat and extend a simple repeating pattern given the core • Find visual patterns in the world around us (e.g., patterns in the rug, in the wallpaper) • Discuss things that repeat in cyclic patterns, (e.g., day and night, days of week) ***Representations:*** • **Model** join and separate situations with objects and pictures ***Symbols:*** • Record mathematical thinking symbolically with teacher assistance	***Patterns and change:*** • Sort objects by one **attribute** and then re-sort by another • Describe the rule used to sort a given a set of pre-sorted objects • Determine the core of the pattern given a set of objects with multiple repetitions (of a simple pattern) • Describe what changes in a repeating pattern ***Representations:*** • **Model** situations in which there is a need to join, separate, compare and use part-part-whole: using objects, pictures, geometric **model**s and symbols ***Symbols:*** • Record mathematical thinking (i.e., invented notation)	***Patterns and change:*** • Sort objects by more than one **attribute** (e.g., boys with glasses, blue squares) • Develop a system for sorting a given set of objects • Create and extend patterns and then translate them into a rule or drawing • Describe the rule for a pattern ***Representations:*** • **Model** situations that involve the addition and subtraction of **whole numbers**, using objects, pictures, geometric **model**s and symbols (e.g., multiplicative thinking may be represented by repeated addition and fair shares by repeated subtraction) ***Symbols:*** • Record mathematical thinking using conventional notation • Use the = sign to connect equivalent parts in a number sentence	***Patterns and change:*** • Find numeric patterns in a hundreds table • Describe the patterns that result when skip-counting ***Representations:*** • **Model** situations that involve the addition, subtraction, and multiplication of **whole numbers** using objects, pictures, symbols, and geometric **model**s ***Symbols:*** • Represent the idea of an unknown quantity using a letter or a symbol • Develop an understanding of the Commutative and Associative properties of whole number addition as a tool to **solve** problems (e.g., is 3+ (7 + 2) always the same as (3 + 7) + 2?)	***Patterns and change:*** • Classify observed patterns of growth according to whether the growth is constant or varied (e.g., plant lab, geometric patterns) • Record patterns of growth in tables and graphs • Interpret tables, graphs and real-world **event**s based on how they change over time ***Representations:*** • **Model** situations that involve the addition, subtraction, multiplication and division of **whole numbers** using objects, pictures, geometric **model**, and symbols ***Symbols:*** • Represent the idea of a variable as an unknown quantity using a letter or symbol • Develop an understanding of the Commutative and Associative Properties of whole number	***Patterns and change:*** • Find a given term in an arithmetic sequence • Translate visual patterns into rules • Describe trends in patterns and graphs ***Representations:*** • **Model** problem situations with objects and use representations such as graphs, tables or equations to draw conclusion ***Symbols:*** • Use equations to express mathematical relationships • Develop an understanding of the Distributive Properties of whole number operations as a tool to **solve** problems (e.g., is 24 × 32 ever the same as 20 × 30 + 4 × 2?)

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				multiplication as a tool to **solve** problems (e.g., is 4 × 5 always the same as 5 × 4?)	

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Enduring Understandings: Change is fundamental to understanding functions. Numbers or objects that repeat in predictable ways can be described or generalized. An operation can be "undone" by its inverse. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
Essential Questions: How can change be described mathematically? How are patterns of change related to the behavior of functions? How do mathematical models/representations shape our understanding of mathematics?

Building upon the K–5 expectations, all students in **Grade 6** will be able to:	Building upon the K–6 expectations, all students in **Grade 7** will be able to:	Building upon the K–7 expectations, all students in **Grade 8** will be able to:	Building upon the K–8 expectations, all students in **Grade 9** will be able to:	Building upon the K–9 expectations, all students in **Grade 10** will be able to:	Building upon the K–10 expectations, all students in **Grade 11** will be able to:
Patterns and change: • Use an expression or rule to describe patterns of change in numeric and geometric patterns ***Representations:*** • Demonstrate that a given situation may be represented by a table, graph or equation • Explore informal methods to **model** and **solve** real-world situations that involve equivalent fractions (e.g., use a table of equivalent ratios to **solve** proportional reasoning problems) • Create a table and **scatter plot** to represent the relationship between two variables ***Symbols:*** • Use **inverse operations** to "do and undo" number sentences	***Patterns and change:*** • Describe rate of change in tables, rules or graphs • Interpret rate of change in tables and graphs based on the context of the problem ***Representations:*** • Connect different representations of the same situation to one another using tables, graphs, and rules • **Model** and **solve** contextualized linear problems using various representations (e.g., tables, graphs, equations) with respect to starting point and rate of change • Describe how the dependent and independent variables are related in a given situation ***Symbols:*** • Write an equation to show how two variables are related • Evaluate an algebraic expression for a given value of the variable	***Patterns and change:*** • Determine the slope of a line given two points on the line (as coordinates, in a graph, in a table) • Use y-intercept and slope to graph the equation of a line • Compare the rates of change in tables and graphs and classify them as linear or nonlinear • Recognize exponential rates of growth and decay in tables and graphs ▪ Use an algebraic expression to represent any term in a numeric or geometric pattern ***Representations:*** • Write an equation given the tabular or graphic form of a linear problem • Analyze the interrelationships among tables, graphs, **and** equations of lines, paying particular attention to the meaning of intercept and slope in the context of the problem	***Patterns and change:*** • **Explain** slope as a rate of change between dependent and independent variables • Compare proportional relationships (direct variation) with linear relationships • Describe the effect of parameter changes on linear and exponential functions • Classify functions as linear or exponential given a table of values, graph, or equation • Compare linear with exponential functions using tables and graphs • Demonstrate a use of recursive thinking to classify linear and exponential functions • Write a linear expression for the general term of an arithmetic sequence and use it to find a specific term ***Representations:*** • **Model** and **solve** real-world linear situations, including linear	***Patterns and change:*** • Use linear, quadratic, and cubic functions to describe length, area, and volume relationships • Compare direct variation with inverse variation • Describe the effect of parameter changes on functions • Analyze the rate of change in exponential and quadratic functions using tables and graphs • Write an expression for the general term of a geometric sequence and use it to find a specific term ***Representations:*** • **Model** and **solve** situations involving systems of equations • Determine the appropriateness of linear, exponential or quadratic **model**s given a real-world situation • Convert flexibly among relationships expressed in tables, graphs, and equations for	***Patterns and change:*** • Use expressions or equations to describe arithmetic and geometric sequences (nth term) and series (sum) • Predict **transformation**s of trigonometric, or inverse functions caused by parameter changes • Use rates of change to classify families of functions • Describe how a change in one variable affects other variables in a multivariable situation ***Representations:*** • Evaluate the advantages and disadvantages of various representations • **Solve** linear inequalities in two variables graphically on a coordinate plane (y>2x+3) • **Model** constraints to **solve** linear programming problems • Analyze linear, quadratic, exponential,

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	• **Solve** linear equations using a variety of **strategies** • Translate visual patterns into symbolic form	• Demonstrate the equivalence of two algebraic expressions using physical **model**s • Use tables, graphs and symbolic reasoning to identify functions as linear or nonlinear ***Symbols:*** • Apply the order of operations • Explore the factor/product relationship between a quadratic expression and its linear factors • Use physical **model**s to develop and write exponential and power **model**s • Combine two algebraic expressions to form a new expression • Demonstrate the equivalence of linear expressions • **Solve** linear equations using **inverse operations** and properties of equality	inequalities, using tables, graphs, and symbols • **Model** and **solve** situations involving systems of equations with tables or graphs using technology • Determine the appropriateness of linear **model**s given a real world situation • Analyze linear and exponential relationships using technology to find an appropriate mathematical **model** • Demonstrate a conceptual understanding of **correlation**. • Demonstrate an understanding of the difference between discrete and continuous data • Convert flexibly among relationships expressed in tables, graphs, and equations for linear functions • Interpret the parameters of a linear or exponential function in terms of the situation it represents • Determine if a given value is a solution to a given equation or inequality	exponential and quadratic functions • Estimate solutions to exponential and quadratic function using tables and graphs • Make strategic selection of graphing calculator viewing window and scale to **solve** problems • Represent and analyze problem situations using **matrice**s ***Symbols:*** • **Solve** systems of linear equations and inequalities algebraically	periodic, trigonometric, or inverse relationships in graphs using best fit lines and curves (regression lines and curve fitting) • Understand the difference and connection between roots of a quadratic equation and factors of a quadratic expression ***Symbols:*** • Use function notation to represent functions • Write equivalent symbolic forms of linear, quadratic, or exponential functions • Use geometric **model**s and algebraic symbols to multiply binomials and complete the square • Use algebraic techniques to identify the vertex and intercepts for quadratic functions • Apply the quadratic formula to **solve** problems

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			Symbols: • Use symbols, tables, and graphs as tools to evaluate expressions • Determine symbolically the equation of a line given combinations of point, slope, and intercept information • Write equivalent forms of linear functions • **Solve** single variable equations and inequalities algebraically		

Standard 3 (K–5) – Geometric Reasoning: Students will develop Geometric Reasoning and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.
Enduring Understandings: Two- and three-dimensional objects can be described, classified, and analyzed by their attributes. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape. An object's location on a plane or in space can be described quantitatively. Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other.
Essential Questions: How are measurement and counting related? How does *what* we measure affect *how* we measure? How can space be defined through numbers/measurement? Why do we compare contrast and classify objects? How do decomposing and recomposing shapes help us build our understand of mathematics? How can transformations be described mathematically?

All students in K**indergarten** will be able to:	Building upon the kindergarten expectations, all students in **Grade 1** will be able to:	Building upon the K–1 expectations, all students in **Grade 2** will be able to:	Building upon the K–2 expectations, all students in **Grade 3** will be able to:	Building upon the K–3 expectations, all students in **Grade 4** will be able to:	Building upon the K–4 expectations, all students in **Grade 5** will be able to:
Classification: • Name and sort figures by shape (e.g., rectangle, triangle, circle) • Recognize **attribute**s and parts of two-dimensional and three-dimensional shapes • Recognize geometric shapes and structures in the environment ***Location and transformation:*** • Find and name locations with simple relationships (e.g., near to, over, under, beside, between, outside, inside) ***Measurement:*** • Compare the length of two objects by placing them side by side • Find items that are longer than or shorter than a given measure (e.g., longer than 10 linker cubes) • Talk about time using calendar (e.g., today, tomorrow and yesterday, and the date)	***Classification:*** • Name and sort plane figures by size and shape • Identify the new shape formed by combining two shapes • Recognize and compare **attribute**s and parts of two-dimensional and three-dimensional shapes ***Location and transformation:*** • Explore symmetry through drawings and use of **manipulatives** • Describe distance in informal terms (e.g., near, far) ***Measurement:*** • Compare the length of two objects by aligning them • Put objects in order according to their length • Compare the weight of two objects using a balance • Use nonstandard units to represent how long an object is	***Classification:*** • Name and sort solid and plane figures by common **attribute**s ***Location and transformation:*** • Recognize shapes that have symmetry ***Measurement:*** • Find objects that are the same in length • Compare the length of two objects by counting the number of nonstandard units used to measure them (e.g., linking cubes) • Balance an object using nonstandard units (e.g., it takes 5 paper clips to balance my pencil) • Measure an object by counting repetitions of the same unit of measure (e.g., the length of the desk measured by an index card) • Measure a large object more than once using a different tool as the unit of measure each time—	***Classification:*** • Name and sort solid and plane figures using several **attribute**s (e.g., number of corners, number of sides, size) • Recognize and represent shapes from different perspectives • Describe, and reason about the results of subdividing and combining shapes ***Location and transformation:*** • Describe a **flip** or **slide** of a given shape that demonstrates that the two shapes are **congruent** • Describe location and movement using geometric vocabulary (e.g., left, right, front, back) • Describe direction of a **turn** using **benchmark turn**s (e.g., 1/4 turn, 1/2 **turn**, full **turn**) ***Measurement:*** • **Explain** the need for standard measurement	***Classification:*** • Identify and classify two-dimensional and three-dimensional shapes according to their properties • Identify and build a three-dimensional object from two-dimensional representation of that object ***Location and transformation:*** • Identify line and **rotation**al symmetry in two-dimensional shapes • Describe a motion or series of motions that will show that two shapes are **congruent** (e.g., **flip**, **slide**, **turn**) • Demonstrate an understanding of **turn rotation** through benchmark angles ***Measurement:*** • Estimate and then measure the length of objects to the nearest whole unit (e.g., find your height in inches or centimeters)	***Classification:*** • Analyze and classify two-dimensional shapes according to their properties and develop definitions for classes of shapes (e.g., a square is a rectangle is a parallelogram is a quadrilateral) • Draw the results of subdividing and combining shapes • Identify and classify angles as acute, right, obtuse, or straight ***Location and transformation:*** • Predict and describe the results of a **slide**, **flip**, or **turn** of two-dimensional shapes • Use the coordinate system to specify locations and to describe paths between locations ***Measurement:*** • Measure and compare objects using standard measures to the nearest 1/2, 1/4 or 1/8th unit

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• Describe and compare volume/capacity of two objects (e.g., full/empty, more/less) • Describe and compare the mass/weight of two objects (e.g., light/ heavy)	• Fill containers using nonstandard units (e.g., water, sand, centimeter cubes) • Talk about the days of the week and the days of the month during calendar time • Recognize coins	decide which one is the "best" for the task • Cover up or "fill in" a design using **manipulatives** (e.g., pattern blocks, color tiles) • Fill up containers and estimate which container hold more • Talk about the time that **event**s happen (e.g., get up, go to lunch, go home, go to bed) • Tell time to the hour • Identify combinations of coin to make one dollar	• Find objects that match a standard unit (e.g., one inch, one foot, one centimeter, one meter) • Measure objects (height, length of arms, length of foot) using standard measurement units (e.g., cm, inches, feet) • Explore what to do if the unit of measure does not work precisely • Make number lines and break each unit into smaller units (e.g., 1/2 units, 1/3 units, 1/4 units) • Find the area of a design by counting the number of units used to cover or fill it (e.g., pattern blocks, color tiles) • Fill up measuring devices (e.g., measuring cups) to informally find volume • Estimate how much time has passed during an **event** • Tell time to the half hour	• Extend the precision of a standard measurement by using fraction strips to develop 1/2, 1/4 or 1/10 as a "unit of measure." • Describe the size of a **turn** angle using clock faces or geo logo • Use a ruler to draw lines or geometric figures with given measurements • Find the distance around a geometric figure to the nearest whole number (perimeter) • Find the number of square units it takes to cover a rectangle (area) • Count the number of cubes it takes to fill a three-dimensional figure (volume) • Use measuring cups and graduated cylinders to find volume • Estimate and then measure the mass of an object to the nearest whole unit • Tell time to the nearest five minutes	• Measure and compare objects using metric units to the nearest $1/10^{th}$ • Use measuring tools to find the size of **turn** angles in degrees • Draw benchmark **turn** angles (30, 45, 60, 90, 180 degrees) • Find the distance around a geometric figure to the nearest 1/2, 1/4 or 1/10th of a unit (perimeter) • Find the number of square units it takes to cover a geometric figure (area) • Find the volume of an object • Find the mass of an object to the nearest 1/2, ¼, or 1/10 of a unit • Convert a measurement from feet to inches, or from meters to centimeters • Find elapsed time

Standard 3 (6–11) – Geometric Reasoning: Students will develop Geometric Reasoning and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.
Enduring Understandings: Two- and three-dimensional objects can be described, classified, and analyzed by their attributes. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape. An object's location on a plane or in space can be described quantitatively. Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other.
Essential Questions: How are measurement and counting related? How does *what* we measure affect *how* we measure? How can space be defined through numbers/measurement? Why do we compare contrast and classify objects? How do decomposing and recomposing shapes help us build our understand of mathematics? How can transformations be described mathematically?

Building upon the K–5 expectations, all students in **Grade 6** will be able to:	Building upon the K–6 expectations, all students in **Grade 7** will be able to:	Building upon the K–7 expectations, all students in **Grade 8** will be able to:	Building upon the K–8 expectations, all students in **Grade 9** will be able to:	Building upon the K–9 expectations, all students in **Grade 10** will be able to:	Building upon the K–10 expectations, all students in **Grade 11** will be able to:
Classification: • Estimate, measure, and classify angles • Identify geometric relationships in the real world (e.g., parallel lines, perpendicular lines, etc.) • Explore the measure of a single angle and find the sum of the angles of a regular polygon ***Location and transformation:*** • Measure angles and sides to demonstrate that **transformation**s such as reflections (**flip**s), **translation**s (**slide**s), and **rotation**s (**turn**s) maintain congruence • Identify the properties of shapes that **tile** a plane ***Measurement:*** • Use the **conceptual knowledge** of the area of rectangles to develop formulas for the areas of triangles and parallelograms • Demonstrate an understanding that the	***Classification:*** • Demonstrate geometric relationships between the measures of angles and sides of triangles and other polygons • Find the measure of the sum of the angles of a closed figure • Build three-dimensional objects from two-dimensional representations and draw two-dimensional representations of three-dimensional objects • Create **model**s of **net**s of three dimensional figures (e.g., cube, rectangular prism, cylinder) ***Location and transformation:*** • Describe the effects that **transformation**s (i.e., **reflection**s, **translation**s, and **rotation**s) and changes in scale have on similarity and congruence • Demonstrate dilations of scale on the coordinate	***Classification:*** • Apply angle relationships to **solve** problems ***Location and transformation:*** • Apply proportional reasoning **strategies** to find unknown sides of similar triangle • Develop and evaluate **mathematical argument**s to demonstrate geometric relationships such as similarity, congruence, or symmetry • Use the Pythagorean Theorem to find missing sides of right triangles ***Measurement:*** • Demonstrate the relationship between the area of the base and volume of prisms and cylinders • Demonstrate the effects of scaling on volume and surface area of rectangular prisms (i.e., how does doubling the side lengths affect the volume?)	***Classification:*** • Represent and **verify** parallel and perpendicular relationships in linear functions • Classify 3-dimensional figures according to the shapes of their base(s) and faces ***Location and transformation:*** • Use properties of polygons to construct them in the coordinate plane ***Measurement:*** • Use appropriate units to measure a given quantity • Analyze precision, accuracy, and approximate error in measurement situations • Apply formulas for area, surface area, and volume of geometric figures including pyramids, cones, spheres, and cylinders • Apply the Pythagorean Theorem	***Classification:*** • Determine whether a triangle is a right triangle (e.g., Converse of Pythagorean Theorem, slopes of adjacent sides) • Identify necessary and sufficient conditions that define parallelograms or triangles • **Justify** whether two figures are similar or **congruent** • Use and **justify** angle relationships created by intersecting and parallel lines • Use **inductive reasoning** to make **conjecture**s and **deductive** reasoning to **justify** conclusions ***Location and transformation:*** • Determine the results of multiple **transformation**s and determine the **transformation**s required to obtain the finished product from the original shape	***Classification:*** • Connect the right angle relationships with the unit circle and periodic functions for any angle • Use Sin and Cos functions to explore periodic real world phenomena • **Justify** whether two triangles are **congruent** using triangle congruence and similarity theorems • Describe and use angle relationships in circles ***Location and transformation:*** • Stretch and shrink periodic functions by changing parameters • Visualize three-dimensional objects from different perspectives and analyze cross-sections ***Measurement:*** • Measure angles in degrees and radians • Convert between degree measures and radian measures

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perimeters of rectangles with a fixed area can vary • Demonstrate an understanding that the areas of rectangles with a fixed perimeter can vary • Find the ratio of the circumference to the diameter of a circular objects to obtain an estimate of π • Use an angle ruler or protractor to measure angles • Sketch a geometric figure given the measure of **turn** angles and the length of sides • Find the circumference of circles and explore finding the area	plane • Represent **reflection**s, **rotation**s, and **translation**s on the coordinate plane ***Measurement:*** • Find the area of polygons by partitioning into rectangles and triangles • Find the surface area of prisms using physical **model**s • Identify corresponding parts of similar figures • Determine the volume and surface areas of cylinders and prisms	• Compare the surface area of rectangular prisms which have the same volume but different dimensions • Find the measures of corresponding parts of similar figures		• Use appropriate technologies to **model** geometric figures and to develop **conjectures** about them • Draw geometric figures in the coordinate plane and **justify** the properties of the figure (e.g., slope, side length) ***Measurement:*** • Apply trigonometric relationships to determine side lengths and angle measures of right triangle • Determine the impact of measurement and rounding error on subsequent computations • Find missing dimensions of a shape given the area, volume, or surface area • Apply the Pythagorean Theorem and its converse • Develop and apply the distance and midpoint formulas • Use partitioning and formulas to find the surface area and volume of complex shapes • Compare the relationship between the volume of different shapes with the same base and height (e.g., cylinder and cone, prism and pyramid)	• Use trigonometric relationships to determine side lengths and angle measures of any triangle

Standard 4 (K–5) – Quantitative Reasoning: Students will develop **Quantitative** Reasoning and an understanding of Data Analysis and Probability by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to **model** mathematical situations to determine the probability.
Enduring Understandings: The question to be answered determines the data to be collected and how best to collect it. Basic statistical techniques can be used to analyze data in the workplace. The probability of an event can be used to predict the probability of future events.
Essential Questions: What is average? What makes a data representation useful? How does my sample affect confidence in my predication? What is fair?

All students in K**indergarten** will be able to:	Building upon the kindergarten expectations, all students in **Grade 1** will be able to:	Building upon the K–1 expectations, all students in **Grade 2** will be able to:	Building upon the K–2 expectations, all students in **Grade 3** will be able to:	Building upon the K–3 expectations, all students in **Grade 4** will be able to:	Building upon the K–4 expectations, all students in **Grade 5** will be able to:
Collect: • Gather and report data about oneself and familiar surroundings using teacher defined categories (preference out of two choices) ***Represent:*** • Use physical objects to organize and informally represent categorical data ***Analyze:*** • Interpret data by making simple comparisons (e.g., more, less, the same) ***Probability:*** • Explore **event**s as likely or unlikely based on shared or personal experiences	***Collect:*** • Collect categorical data (observe and count frequencies) to answer a question posed by the teacher ***Represent:*** • Organize and informally represent categorical data (2 or 3 categories) using drawings or physical objects ***Analyze:*** • Interpret data by making comparisons between frequencies of categorical data (e.g., how many more) ***Probability:*** • Explore **event**s as likely or unlikely, possible or impossible based on shared or personal experience	***Collect:*** • Collect (e.g., observe, count, or survey) categorical data to answer a question posed by the teacher or students ***Represent:*** • Demonstrate a variety of informal techniques for organizing and representing categorical data (e.g., tallies, pictures, or physical objects, bar graph with scale provided, **line plot**) ***Analyze:*** • Interpret data by noting characteristics of the graph (e.g., most, least, the same) ***Probability:*** • Explore **event**s as more likely or less likely based on informal observation	***Collect:*** • Collect categorical and numerical data to answer a question posed by the teacher or students ***Represent:*** • Demonstrate a variety of informal and conventional techniques for representing and organizing categorical and numerical data (e.g., tallies, tables, **pictograph**s, bar graphs) ***Analyze:*** • See and describe data as a whole, describing the shape of the distribution; reason about how individual pieces of data relate to the whole • Find and use the mode to describe and interpret data ***Probability:*** • Describe the likelihood of an **event** based on experimental observations using simple randomizing devices (e.g. spinners, number cubes) and	***Collect:*** • Pose questions that can be answered with data; systematically collect and organize both categorical and numerical data • Collect categorical data where the data is described using numbers (e.g., how many have five letters in their first name?) ***Represent:*** • Construct and use data displays (e.g., tables, scaled **pictograph**s, bar graphs, **line plot**s) in order to answer a question ***Analyze:*** • Describe a set of data as a whole, noting important features such as concentration of values, spread of the values, and extreme values • Find and use measures of center (mode and median) to summarize and interpret data ***Probability:*** • Describe the outcomes of an experiment or	***Collect:*** • Pose questions that can be answered with data; systematically collect and organize categorical and numerical/ measurement data ***Represent:*** • Construct and use data displays (e.g., tables, scaled **pictograph**s, **line plot**s, bar graphs) in order to answer a question ***Analyze:*** • Compare related data sets noting similarities and differences in the distributions • Find and use measures of center (mean, median, mode) and spread (range) to summarize and interpret data • Identify the typical or average value in a data set as well as any atypical values ***Probability:*** • Conduct a probability experiment, represent the result as a number (fraction, decimal, percent) between 0 and

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			ideas such as certain, impossible, and equally likely	**event** (e.g., possible/ impossible, certain/ uncertain, less likely/ equally likely/more likely)	1, and draw conclusions from the results • List all possible outcomes (i.e. the **sample space**) for a probability experiment involving a simple **event**

Standard 4 (6–11) – Quantitative Reasoning: Students will develop **Quantitative** Reasoning and an understanding of Data Analysis and Probability by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to **model** mathematical situations to determine the probability.
Enduring Understandings: The question to be answered determines the data to be collected and how best to collect it. Basic statistical techniques can be used to analyze data in the workplace. The probability of an event can be used to predict the probability of future events.
Essential Questions: What is average? What makes a data representation useful? How does my sample affect confidence in my predication? What is fair?

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Collect: • Collect and organize numerical (whole number or decimal) data in order to answer a question ***Represent:*** • Construct displays of data (e.g., circle graphs, **scatter plot**s, frequency counts) for a single data set ***Analyze:*** • Defend conclusions drawn from the interpretation of data by comparing one data set to another • Find and use summary measures of center (mean, median, mode) and spread (range) to compare sets of single variable data ***Probability:*** • Use real-world data to estimate the probability of future **event**s (e.g., batting averages, weather predictions) • Analyze simple games to determine favorable outcomes and chances of winning or losing	***Collect:*** • Pose questions that can be answered by collecting and organizing data from experiments, surveys, and relevant print and electronic resources ***Represent:*** • Construct displays of data for single data sets (e.g., **stem-and-leaf plot**s) or in order to study the relationship between related data sets (**scatter plot**s) ***Analyze:*** • Defend or dispute conclusions drawn from the interpretation of data by comparing one data set to another • Choose an appropriate measures of center (mean, median, mode) and spread (range) to interpret data set(s) ***Probability:*** • Construct a **sample space** (organized list, counting tree) to determine theoretical probabilities of an **event** • Use proportional reasoning to predict how often a simple	***Collect:*** • Pose questions that can be answered by collecting and organizing data from experiments, surveys, and relevant print and electronic resources • Use random sampling methods to collect the necessary information to answer questions ***Represent:*** • Construct displays of data to represent individual sets of data (e.g., **histogram**s, **box plot**s) or to explore the relationship between related sets of data (**scatter plot**s, line graphs); describe the correspondence between data sets and their graphical displays ***Analyze:*** • Defend or dispute conclusions drawn from the interpretation of data by comparing sets of data or exploring possible relationships based upon **scatter plot**s of related data and approximate lines of fit	***Collect:*** • Describe and **explain** how the validity of predictions are affected by number of trials, sample size, and the population ***Represent:*** • Select and interpret the most appropriate display for a given purpose and set(s) of data (e.g., **histogram**s, parallel **box plot**s, **stem-and-leaf plot**s, **scatter plot**s) • Find an appropriate mathematical **model** of a linear or exponential function and use the **model** to make predictions recognizing the limitations of the **model** ***Analyze:*** • Analyze the validity of statistical conclusions on both one- and two-variable data. • Describe the effect of outliers in both one-variable and two-variable contexts ***Probability:*** • Use and design **simulation**s or	***Collect:*** • Use permutations and combinations as counting techniques ***Represent:*** • N/A ***Analyze:*** • Recognize how linear **transformation**s of one variable data affect shape, center, and spread ***Probability:*** • Compute and interpret expected value • Compute the probability of both independent and **dependent events**	***Collect:*** • Understand the differences among the various kinds of studies (e.g., survey, controlled experiment) • Determine factors which may affect the outcome of a survey • Design surveys and apply random sampling techniques to avoid bias in the data collection ***Represent:*** • Interpret least squares regression line as the line that minimizes the sum of the squared errors ***Analyze:*** • Apply benchmark percents in a normal distribution • Compute and use standard deviation to analyze data variability • Recognize approximately norm distributions • Interpret margin of error and confidence intervals ***Probability:*** • Identify mutually exclusive **event**s and apply the addition rule to

Building upon the K–5 expectations, all students in **Grade 6** will be able to:	Building upon the K–6 expectations, all students in **Grade 7** will be able to:	Building upon the K–7 expectations, all students in **Grade 8** will be able to:	Building upon the K–8 expectations, all students in **Grade 9** will be able to:	Building upon the K–9 expectations, all students in **Grade 10** will be able to:	Building upon the K–10 expectations, all students in **Grade 11** will be able to:
	probability **event** will occur in a given number of trials	• Analyze a representative sample to make inferences about a population • Find and use appropriate measures of center (mean, media, mode) and spread (range, **interquartile range**) to interpret data • Compare the usefulness of the mean and median as measures of center; describe the effect of changes in the data on the mean and median of the data set(s) ***Probability:*** • Compare and make predictions based on theoretical and experimental probabilities, using sample data generated through actual experiments or computer simulations • Construct an appropriate **sample space** and apply principles of probability for a simple or compound **event** • Investigate and describe the difference between the **event experimental probability** of a simulated **event** (experiment) and the **theoretical probability** of the same **event** • Explore the concepts of randomness and random sample	experiments to determine probabilities of independent and **dependent events** • Define a **sample space** to compare probabilities using the **Fundamental Counting Principle** • Compare **event experimental probability** with **theoretical probability** (Law of Large Numbers)		compute the probability of the occurrence

Standard 5, 6, 7, and 8 – Process Standards
Enduring Understandings: Mathematics can be used to solve problems outside of the mathematics classroom. Mathematics is built on reason and always makes sense. Reasoning allows us to *make* conjectures and to *prove* conjectures. Classifying helps us build networks of mathematical ideas. Precise language helps us express mathematical ideas and receive them.
Essential Questions: Is your plan working? Do you need to reconsider what you are doing? How are *solving* and *proving* different? How are *showing* and *explaining* different? How do you know when you have proven something? What does it take to **verify** a conjecture? How do you develop a convincing argument? How do you make sense of different strategies? How do you determine their strengths and weaknesses? How do you determine similarities and differences? Why do we classify? Why do we classify numbers? Why do we classify geometric objects? Why do we classify functions?

Standard 5 – Problem Solving: Students will develop their Problem Solving ability by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply **strategies** to **solve** a wide variety of problems; and to integrate mathematical reasoning, communication and connections. All students in grades K–12 will be able to:

- Build new **mathematical knowledge**
- **Solve** problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate **strategies** to **solve** problems
- Monitor and reflect on the process of mathematical problem solving

Standard 6 – Reasoning and Proof: Students will develop their Reasoning and **Proof** ability by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to **justify** their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking; and to construct their own learning. All students in grades K–12 will be able to:

- Understand that reasoning and **proof** are fundamental aspects of mathematics
- Make and investigate mathematical **conjectures**
- Develop and evaluate **mathematical argument**s and **proof**s
- Select and use various types of reasoning and methods of **proof**

Standard 7 – Communication: Students will develop their mathematical Communication ability by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats. All students in grades K–12 will be able to:

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and **strategies** of others
- Use the language of mathematics to express mathematical ideas precisely

Standard 8 – Connections: Students will develop mathematical Connections by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways. All students in grades K–12 will be able to:

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

GLOSSARY

A

algorithm: A rule or procedure for computing or solving a certain type of problem.

attribute: A distinct feature or property.

average: A number that represents a set of data, typically the mean, median, or mode.

B

benchmark: A commonly used point of reference for a measurement or comparison.

box plot: A graph that summarizes data using the median, upper and lower quartiles, and the extreme values. A box is drawn around the quartile values and the whiskers extend from each quartile to the extreme data points.

C

conceptual knowledge: Understanding of mathematical ideas that are abstracted through repeated experiences and reflection.

congruent: Geometric figures having the same shape, size and measurements.

cooperative learning: Students working together in small groups to solve problems in which collaboration is helpful and useful.

conjecture: A generalization or hypothesis made by observing data and recognizing patterns (inductive reasoning) without sufficient evidence for proof.

content standards: Statements about what it is that students should know and be able to do. They indicate the most enduring and important knowledge and skills essential to a discipline that students are expected to learn.

correlation: The degree of relative correspondence between two sets of data.

D

decompose: To separate into constituent parts, i.e. 18 = 12 + 6.

deductive (logical) reasoning: Process of demonstrating that if certain statements (axioms, postulates, theorems) are accepted as true, then other statements can be shown or proved to follow from them.

dependent events: Two events for which the occurrence of one affects that of the other.

E

event: In probability, any possible realistic outcome.

experimental (empirical) probability: A probability formulated using the results of an experiment or series of trials.

explain: To give the reason for a calculation or to make one's mathematical thinking understandable.

F

flip: See reflection.

functional relationship: A relationship that pairs each object in a given set with exactly one object in a second set. (e.g., the area of a circle is a function of the radius - if the radius is 4", the area is 16^1 sq.")

fundamental counting principle: If one event can occur in *n* ways and, for each of those, a second event can occur in *m* ways, then the two events can occur in *mn* ways.

H

histogram: A graphic picture of a frequency distribution using rectangles with class intervals as the bases and areas proportional to the frequency of that interval. The intervals include all possible values of the data; therefore there are no spaces between the bars of the graph.

I

independent events: Two events for which the occurrence of the one event does not affect that of the other.

inductive reasoning: Process of observing data, recognizing patterns, and making generalizations from the observations.

integers: The numbers in the set {...,-3, -2, -1, 0, 1, 2, 3, ...}.

interquartile range: The range of data within the "box" of a box plot determined by subtracting the lower quartile from the upper quartile.

inverse operations: Operations that undo each other; addition and subtraction are inverse operations.

irrational number: A real number that cannot be written as the quotient of two integers, i..e., π or $\sqrt{2}$; a non-repeating, non-terminating decimal.

J

justify: Connecting one's own thinking to a mathematical concept in order to make the explanation or argument convincing.

L

line plot: A graph showing the frequency of data on a number line.

M

manipulatives: Concrete models useful in representing various mathematical concepts. They are used to experiment with and explore various mathematical ideas.

mathematical argument: A mathematical explanation that proves or disproves an hypothesis.

mathematical knowledge: The context in which the mathematical processes are used.

matrice: A rectangular array of terms written between parentheses or double lines on either side of the array. A matrix does not have a quantitative value.

mean: A way to describe the "middle" of a set of data that is found by adding all of the values and dividing by the number of values.

measures of central tendency: The mean, median, and mode of a set of data.

median: A value that is halfway through an ordered set of data. If there are an even number of values, the median is the value that is halfway between the middle pair of values.

mode: The most frequently occurring value in a set of data. A data set may have more than one mode.

model: A process in which students represent problem situations mathematically using concrete, oral, written, pictorial, graphic, numeric and/or algebraic methods.

N

net: A two-dimensional pattern for a three-dimensional object.

number sense: Having well-understood number meanings; having knowledge of multiple relationships among numbers; recognizing the relative magnitudes of numbers; knowing the relative effect of operating on numbers; and developing referents for measures of common objects and situations in their environments.

O

outlier: Data values so large or small that they stand apart from the rest of the distribution.

P

partitioning: Dividing into parts.

pictograph: A graph in which a symbol is used to represent a given number of items.

process standards: Statements about what it is that students engage in when acquiring and using mathematics content knowledge - problem solving, reasoning, communicating and connecting.

proof: A logical argument or chain of reasoning based on previously accepted facts and assumptions used to conclusively defend a mathematical statement or generalization.

Q

quantitative: Relating to measuring and counting data.

quartile: One of four equal divisions of a set of data.

R

range: The difference between the largest and smallest values in a set of data.

rational number: A real number that can be written as a quotient of two integers a/b, where b does not equal 0; a repeating or terminating decimal.

real number: Any number that is either rational or irrational.

recompose: Put together again, (i.e. 12+ 6 = 18 and 11 + 7 = 18).

recursive: A pattern in which each term can be generated by applying a formula to the preceding term

reflection: Replacing each point in a figure by a point symmetric with respect to a line. (Flipping a figure across a line)

rotation: Movement of a figure in a circular motion about a point. (Turning an object about a point)

S

sample space: The set or collection of all possible outcomes of a probability experiment.

Scatter plot: A graph consisting of ordered pairs possibly showing a relationship between two variable quantities.

simulation: The process of modeling the outcomes of a particular experiment in probability; often manipulated through computers.

slide: See translation.

solve: To find all values of a variable that make a sentence true.

stem-and-leaf plot: A method of organizing data using the digits of the greatest place value to group the data.

strategies: Methods used to solve problems.

symmetry: A figure has symmetry if it remains the same after some operation or transformation is done to it.

T

theoretical (a priori) probability: A probability formulated using prior or intuitive knowledge of the sample space before an experiment is performed.

tile: To use a repeating pattern of congruent figures that fit together with no gaps and no overlaps

transformation: Process of changing the position, size, or shape of a figure according to a given rule such as rotation, translation, dilation, and reflection.

translation: Changing the coordinates of points to coordinates referred to new axes parallel to the old. (Sliding a figure from one location to another)

turn angle: The angle of rotation.

turn: See rotation.

V

verify: To demonstrate the truth or accuracy of one's mathematical thinking.

W

whole numbers: The numbers in the set {0, 1, 2, 3, ...}.