

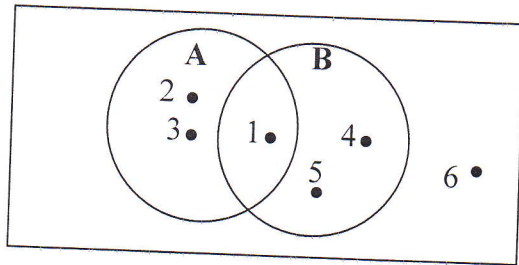
6pts 1. a) How many 3-person committees can be formed using a group of seven people if one is the president, another is the secretary, and the third the treasurer?

$$7P_3 = \frac{7!}{4!} = \boxed{210}$$

b) Evaluate $\frac{948! 534!}{947! 535!}$

$$\frac{948}{535}$$

6pts 2. The accompanying Venn diagram describes the sample space of a particular experiment and events A and B. Suppose $P(1) = P(2) = P(3) = 0.20$, $P(4) = 0.15$ and $P(5) = 0.05$ and $P(6) = 0.20$. Find $P(A)$ and $P(B)$.



$$P(A) = 0.2 + 0.2 + 0.2 = \boxed{0.60}$$

1, 2, 3

$$P(B) = 0.20 + 0.15 + 0.05 = \boxed{0.40}$$

1, 4, 5

6pts 3. The following table describes the adult population of a large city of 300,000 people

- A: {person under 21 years}
- B: {person 21-50 years}
- C: {person over 50 years}
- D: {income less than 40k}
- E: {income 40k - 100k}
- F: {income over 100k}

	D under 40k	E 40k-100k	F over 100k	
A under 21	18,000	64,000	30,000	112,000
B 21-50yrs	24,000	36,000	46,000	106,000
C over 50	20,000	24,000	38,000	82,000
	62,000	124,000	114,000	300,000

a) Find $P(A \cup F)$

$$\frac{112,000 + 46,000 + 38,000}{300,000}$$

$$\frac{196}{300} = \boxed{\frac{49}{75}}$$

.653

b) Find $P(C \cap D)$

$$\frac{20,000}{300,000}$$

$$\frac{20}{300} = \boxed{\frac{1}{15}}$$

.067

c) Find $P(B^c)$

$$\frac{112,000 + 82,000}{300,000}$$

$$\frac{194}{300} = \boxed{\frac{97}{150}}$$

.647

9pts 4. Given a standard deck of 52 cards

a) Find the probability of drawing a black face card.

$$\frac{6}{52}$$

b) Find the probability of drawing a number card.

$$\frac{36}{52}$$

c) Find the probability of drawing a red ace or a card that is not a diamond.

$$\frac{2}{52} + \frac{39}{52} - \frac{1}{52} = \frac{40}{52}$$

10pts 5. Suppose the events A and B from a die toss experiment are as follows

A: {Odd number is rolled}

$$\{1, 3, 5\} \quad P(A) = \frac{3}{6}$$

$$A \cap B = \{5\}$$

B: {Number more than 4 is rolled}

$$\{5, 6\} \quad P(B) = \frac{2}{6}$$

$$P(A \cap B) = \frac{1}{6}$$

a) Find the conditional probability $P(A/B)$

$$\frac{P(A \cap B)}{P(B)} = \frac{1/6}{2/6} = \frac{1}{2}$$

b) Are the events A and B independent or dependent? Show work to support answer.

$$P(A/B) = P(A) \\ \frac{1}{2} = \frac{3}{6} \checkmark$$

They are independent
Since $P(A/B) = P(A)$

6pts 6. Is the random variable involved continuous or discrete?

a) The number of cars currently in the parking lot.

Discrete

b) The time it takes to complete this test.

Continuous

c) The number of students who receive a C or better on this test.

Discrete

10pts 7. Four coins are tossed. Let $x = \#$ heads observed.

a) Identify the 16 simple events associated with this experiment and assign a value of x to each.

HHHH	4	HHTT	2	THHH	3	TTHH	2
HHHT	3	HHTH	3	THHT	2	TTHT	1
HTHH	3	HHTH	2	THTT	1	TTTH	1
HTHT	2	HTTT	1	THTH	2	TTTT	0

b) Display the probability distribution of x in tabular form.

x	0	1	2	3	4
$P(x)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

10pts 8. Given the following probability distribution for the random variable x , find $E(x)$ and σ .

x	30	50	70	100	120
$p(x)$	0.10	0.20	0.10	0.40	0.20

$$E(x) = (30)(0.10) + \dots + (120)(0.20) = \boxed{84}$$

$$\sigma = \sqrt{(84-30)^2(0.10) + \dots + (120-84)^2(0.20)}$$

$$= \sqrt{291.6 + 231.2 + 19.6 + 102.4 + 259.2} = \sqrt{904} = \boxed{30.0666}$$

In problems 9-12 Identify the distribution (Binomial, Poisson, Geometric, Hypergeometric). Also identify (λ , x , n , p , etc) Then use the tables in the appendix and/or the formulas discussed in class to find the probability

8pts 9. If eight cards are drawn without replacement from a standard deck of 52 cards what is the probability that there are either 1 or 2 faces?

Hypergeometric

$$\frac{\binom{12}{1}\binom{40}{7} + \binom{12}{2}\binom{40}{6}}{\binom{52}{8}} = \frac{\binom{12}{1}(18643560) + \binom{66}{2}(3838380)}{752,538,150}$$

$$= \frac{477055800}{752,538,150} = \boxed{0.63393}$$

8 pts 10. Suppose 4 red marbles, 9 white marbles, and 7 blue marbles are placed in a bag. You draw a marble and then replace it before another is drawn. What is the probability of getting the first red marble on the fourth draw?

$p = \frac{4}{20} = .2$
 $q = \frac{16}{20} = .8$

$(.8)^3 (.2)^1$

Geometric

$.1024$

8pts 11. Suppose 12 black marbles, 6 red marbles and 2 green marbles are placed in a bag. You draw a marble and then replace it before another is drawn. What is the probability of getting at least seven black marbles from fifteen draws?

$n=15$
 $p = \frac{12}{20} = .6$
 $q = .4$
Binom.

$P(X \geq 7)$
 $1 - P(X \leq 6)$
 $1 - .095$
 0.905

$\binom{15}{0} (.6)^0 (.4)^{15} \approx 0$
 $\binom{15}{1} (.6)^1 (.4)^{14} \approx 0$
 $\binom{15}{2} (.6)^2 (.4)^{13} \approx .0003$
 $\binom{15}{3} (.6)^3 (.4)^{12} = .0016$
 $\binom{15}{4} (.6)^4 (.4)^{11} = .0074$
 $\binom{15}{5} (.6)^5 (.4)^{10} = .0245$
 $\binom{15}{6} (.6)^6 (.4)^9 = .0612$
 $1 - .095$
 $\frac{\lambda^x e^{-\lambda}}{x!}$

8pts 12. A company reports the number of breakdowns per day is on the average 3.6. What is the probability on a given day that five or fewer breakdowns would occur?

Poisson
 $\lambda = 3.6$

$P(X \leq 5)$
 $.844$

$P(0) = \frac{(3.6)^0 e^{-3.6}}{0!} = .0273$
 $P(1) = \frac{(3.6)^1 e^{-3.6}}{1!} = .0984$
 $P(2) = \frac{(3.6)^2 e^{-3.6}}{2!} = .17705$
 $P(3) = \frac{(3.6)^3 e^{-3.6}}{3!} = .21247$
 $P(4) = \frac{(3.6)^4 e^{-3.6}}{4!} = .19122$
 $P(5) = \frac{(3.6)^5 e^{-3.6}}{5!} = .13768$