MATH221 CALCULUS I 100pts TEST 4 Spring'11 INSTRUCTOR: CARLA MORRIS

Page 1 NAME: Key

x^{-3}

6pts 1. Find $\int (e^{7x} + 6x^2 + 2x + 1/x)\,dx$

$\boxed{\frac{e^{7x}}{7} + 2x^3 + x^2 + \ln|x| + C}$

6pts 2. Find $\int (9x^8 + 12x^3 + 1/x^3)\,dx$

$\boxed{x^9 + 3x^4 - \frac{1}{2x^2} + C}$

6pts 3. Find a function with the following properties $f'(x) = (7/3)x^{4/3}$ and $f(1) = 5$.

$f(x) = \frac{\cancel{7/3}\, x^{7/3}}{\cancel{7/3}} + C$

$f(1) = 1 + C = 5 \quad C = 4$

$\boxed{f(x) = x^{7/3} + 4}$

6pts 4. Find the fourth Riemann sum of $3x + 4$ on $[3, 23]$ using left endpoints.

$\frac{23-3}{4} = 5$

left ends

3, 8, 13, 18

$5[f(3) + f(8) + f(13) + f(18)]$

$5[13 + 28 + 43 + 58] = 5[142] = \boxed{710}$

6pts 5. Find $\int_1^4 (5x^4 + 3x^2 + 4x + 2)\,dx$

$(x^5 + x^3 + 2x^2 + 2x)\Big|_1^4$

$1024 + 64 + 32 + 8 - 6$ $\boxed{1122}$

6pts 6. Find $\int_1^5 9e^{3x}\,dx$

$\frac{9e^{3x}}{3}\Big|_1^5$ $\boxed{3e^{15} - 3e^3}$

6pts 7. Find the area of the region bounded by the curves $y = x^2 - 5x + 6$ and $y = 2x - 4$

3

$x^2 - 5x + 6 = 0$

$2x - 4 = 2$

$x^2 - 5x + 6 = 2x - 4$

$x^2 - 7x + 10 = 0$ $\quad 2, 5$

$(x-5)(x-2) = 0$

$\int_2^5 [(2x-4) - (x^2 - 5x + 6)]\,dx$

$\int_2^5 (-x^2 + 7x - 10)\,dx$

$\left(-\frac{x^3}{3} + \frac{7x^2}{2} - 10x\right)\Big|_2^5$ $\boxed{9/2}$

$\left(-\frac{125}{3} + \frac{175}{2} - 50\right) - \left(-\frac{8}{3} + 14 - 20\right)$

$-\frac{250}{6} + \frac{525}{6} - \frac{300}{6} + \frac{16}{6} - \frac{84}{6} + \frac{120}{6} = \frac{27}{6}$

6pts 8. Find the consumer surplus for the demand at the given sales level x $p = 3 - (x/10)$ at $x = 20$

$\int_0^{20} [(3 - x/10) - 1]\,dx$

$\int_0^{20} (2 - x/10)\,dx$

$2x - \frac{x^2}{20}\Big|_0^{20}$

$(40 - 20) - 0$

\$20

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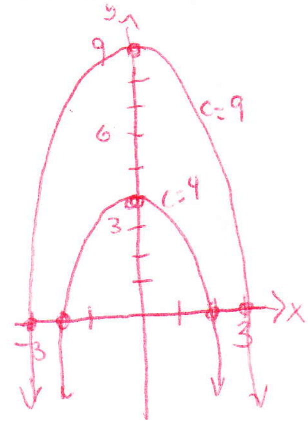
6pts 9. If $f(x,y) = 7x^5 - 6x^3y + 3x^2 + 7y - 3$ find $f(1, 2)$

$7(1)^5 - 6(1)^3(2) + 3(1)^2 + 7(2) - 3$ $\boxed{9}$

6pts 10. Sketch level curves for $f(x, y) = 4$ and $f(x, y) = 9$ if $f(x, y) = x^2 + y$

$x^2+y=4$ $y=4-x^2$ $y=9-x^2$



10pts 11. Use a second derivative test for bivariates to determine all local extrema for
$f(x, y) = 3x^2 - 4xy + 3y^2 + 8x - 17y + 30$

$f_x = 6x - 4y + 8$

$f_y = -4x + 6y - 17$

$f_{xx} = 6$

$f_{yy} = 6$

$f_{xy} = f_{yx} = -4$

poss extrema $(1, 7/2)$

$D(1, 7/2) = (6)(6) - (-4)^2 > 0$

$f_{xx} > 0$

$2(6x - 4y = -8)$

$3(-4x + 6y = 17)$

$12x - 8y = -16$

$-12x + 18y = 51$

$10y = 35$

$y = 7/2$ $x = 1$

$\boxed{\text{local min } f(1, 7/2) = 17/4}$

6pts 12. Find $\int (12x^2 + 20x)(2x^3 + 5x^2 + 5)^9 \, dx$

$u = 2x^3 + 5x^2 + 5$

$du = (6x^2 + 10x)dx$

$2du = (12x^2 + 20x)dx$

$\int u^9 \, 2du$

$\frac{2u^{10}}{10} + C$

$\frac{u^{10}}{5} + C$

$\boxed{\frac{(2x^3+5x^2+5)^{10}}{5} + C}$

6pts 13. Find $\int xe^{8x} \, dx$

$f(x) = x$ $f'(x) = 1dx$

$g'(x) = e^{8x}$ $g(x) = \frac{e^{8x}}{8}$

$\frac{xe^{8x}}{8} - \int \frac{e^{8x}}{8} dx$

$\boxed{\frac{xe^{8x}}{8} - \frac{e^{8x}}{64} + C}$

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6pts 14. Find $\int (x^{20}\ln 3x)dx$

$f(x) = \ln 3x$ $f'(x) = \frac{1}{x}dx$

$g'(x) = x^{20}$ $g(x) = \frac{x^{21}}{21}$

$\frac{x^{21}}{21}\ln 3x - \int \frac{x^{20}}{21}dx$

$x^{21}/21 \ln 3x - \frac{x^{21}}{441} + C$

6pts 15. Find $\int 8x\sqrt{x^2-9}\,dx$

$u = x^2-9$

$4du = 8x\,dx$

$\int 4u^{1/2}du$

$\frac{4u^{3/2}}{3/2} + C$

$\boxed{\frac{8}{3}(x^2-9)^{3/2} + C}$

6pts 16. Find $\int 8x^3e^{x^4}dx$

$u = x^4$

$2\,du = 8x^3dx$

$\int e^u\,2du$

$\boxed{2e^{x^4} + C}$

FORMULAS

1. $\frac{b-a}{n} \Sigma f(x_i)$

2. $\int [f(x) - B]dx$

3 $\int_0^{x_0} [p(x) - p_0]dx$

4. $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)\,dx$

5. $d/dx[\ln g(x)] = \frac{g'(x)}{g(x)}$

6. $d/dx[e^{g(x)}] = e^{g(x)}g'(x)$

BIVARIATE SECOND DERIVATIVE TEST

Let f be a bivariate function and suppose $f_x(a,b) = 0$ and $f_y(a,b) = 0$

Let $D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a) If $D > 0$ and $f_{xx}(a,b) > 0$ then $f(x,y)$ has local min at (a,b)

b) If $D > 0$ and $f_{xx}(a,b) < 0$ then $f(x,y)$ has local max at (a,b)

c) If $D < 0$ then $f(x,y)$ does not have an extremum at (a,b)

d) If $D = 0$ then test is inconclusive