MATH221 CALCULUS I 100pts TEST 3 Spring'11 INSTRUCTOR: CARLA MORRIS
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In problems 1-4 differentiate (5pts each) you can leave answer after first step

1. $f(x) = (5x^7 + 2x^6 + 9x^3)^5 (7x+3)^8$

$(5x^7+2x^6+9x^3)^5 [8(7x+3)^7(7)]$
$+ (7x+3)^8 [5(5x^7+2x^6+9x^3)^4(35x^6+12x^5+27x^2)]$

2. $f(x) = \dfrac{9}{(5x^2+8x+1)^4}$ $\frac{d}{dx}[9(5x^2+8x+1)^{-4}]$

$-36(5x^2+8x+1)^{-5}(10x+8)$

3. $f(x) = (7x^4 + 7x^2 + 1/x^4)^{25}$

$25(7x^4+7x^2+\frac{1}{x^4})^{24}\left[28x^3+14x-\frac{4}{x^5}\right]$

4. $f(x) = \left(\dfrac{x^2+8x+5}{x^3+27x+1}\right)^{20}$

$20\left(\frac{x^2+8x+5}{x^3+27x+1}\right)^{19}\left[\frac{(x^3+27x+1)(2x+8)-(x^2+8x+5)(3x^2+27)}{(x^3+27x+1)^2}\right]$

5pts 5. Find dy/dx using the chain rule if $y = 7u^8$ and $u = 4x^5 + 2x^3 + 1$

$\frac{dy}{du} = 56u^7$

$\frac{du}{dx} = 20x^4+6x^2$

$56u^7(20x^4+6x^2)$

$56(4x^5+2x^3+1)^7(20x^4+6x^2)$

5pts 6. Find the equation of the tangent line to the curve $y = (3x-5)^4(x+9)$ at $x = 2$ pt(2, 11)

$[(3x-5)^4(1)+(x+9)4(3x-5)^3(3)]\big|_{x=2}$

$(1)(1) + 11(4)(1)(3) = 133 = m$

$y - 11 = 133(x-2)$

IN PROBLEMS 7-8 USE IMPLICIT DIFFERENTIATION TO FIND dy / dx

5pts 7. $4x^6 - 6y^3 = 9x^3 + 7y$

$24x^5 - 18y^2\frac{dy}{dx} = 27x^2 + 7\frac{dy}{dx}$

$24x^5 - 27x^2 = (18y^2+7)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{24x^5-27x^2}{18y^2+7}$

5pts 8. $3x^6y^5 + 7x^3 = 9y^2 + 18x$

$3x^6(5y^4\frac{dy}{dx}) + y^5(18x^5) + 21x^2 = 18y\frac{dy}{dx} + 18$

$(15x^6y^4 - 18y)\frac{dy}{dx} = 18 - 21x^2 - 18x^5y^5$

$\frac{dy}{dx} = \frac{18-21x^2-18x^5y^5}{15x^6y^4-18y}$

5pts 9. Solve for x: $4^{5x+8} = (8)(4^{2x+5})$

$2^{10x+16} = 2^3 2^{9x+10}$

$10x+16 = 9x+13$

$6x = -3$

$x = -\frac{1}{6}$

4pts 10. Solve for x : $(5^x)(x^3) - (5^x)(13x^2) + (5^x)(42x) = 0$

$(5^x)(x)(x^2-13x+42)=0$

$(x)(5^x)(x-6)(x-7)=0$

$5^x \neq 0$

$x = 0, 6, 7$

4pts 11. Solve for x : $e^{4x} = 625$

$\ln e^{4x} = \ln 625$

$4x = \ln 625$

$x = \frac{1}{4}\ln 625$

$x = \ln 5$

IN PROBLEMS 12-16 DIFFERENTIATE THE GIVEN FUNCTIONS (5PTS EACH)

12. $f(x) = 20x^5 e^{7x}$

$20x^5(7e^{7x}) + e^{7x}(100x^4)$

13. $f(x) = (8x^6 + 4x^5 + 8x + 1)^6 (e^x)$

$(8x^6+4x^5+8x+1)^6 e^x + e^x(6)(8x^6+4x^5+8x+1)^5 (48x^5+20x^4+8)$

14. $f(x) = \dfrac{e^{4x}}{(2x^2-5x+3)^8}$

$\dfrac{(2x^2-5x+3)^8 4e^{4x} - e^{4x}(8)(2x^2-5x+3)^7(4x-5)}{(2x^2-5x+3)^{16}}$

15. $f(x) = \ln(7x^6 + 7x^4 + 8x^3 + 9x + 8)$

$\dfrac{42x^5+28x^3+24x^2+9}{7x^6+7x^4+8x^3+9x+8}$

16. $f(x) = (7x^2 + 4x + 3)^{11} \ln 5x$

$(7x^2+4x+3)^{11}\left(\frac{1}{x}\right) + \ln 5x\,(11)(7x^2+4x+3)^{10}(14x+9)$

4pts 17. Write as a single logarithm $3\ln(4x-5) - 5\ln(3x+1) - 2\ln(5x+3)$

$\ln(4x-5)^3 - \ln(3x+1)^5 - \ln(5x+3)^2$

$\ln\left[\dfrac{(4x-5)^3}{(3x+1)^5(5x+3)^2}\right]$

4pts 18. Differentiate $\ln[(3x-5)^4(5x^3+9x+4)^{10}]$ by writing as simpler logs first.

$4\ln(3x-5) + 10\ln(5x^3+9x+4)$

$4\left(\frac{3}{3x-5}\right) + 10\left(\frac{15x^2+9}{5x^3+9x+4}\right)$

5pts 19. Suppose \$750,000 is invested at 6% compounded quarterly for 12 years. How much will it be worth at the end of that time?

$750{,}000\left(1+\frac{.06}{4}\right)^{4\cdot 12}$

$750{,}000(1.015)^{48} = \$1{,}532{,}608.72$

5pts 20. Suppose \$500,000 is invested at 3% compounded continuously for 24 years. How much interest does it earn at the end of that time ?

$500{,}000\,e^{.03(24)} - 500{,}000$

$1{,}027{,}216.61 - 500{,}000$

\$527,216.61 interest

4pts 21. Determine the percentage rate of change of the function at the point indicated
$f(t) = 6t^5$ at $t = 2$

$\frac{30t^4}{6t^5} \times 100\%$ $\frac{5}{2} \times 100\%$ 250%

FORMULAS

1. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
2. $\frac{f'(t)}{f(t)} \times 100\%$
3. $I = Prt$
4. $A = P(1 + r/n)^{nt}$
5. $A = Pe^{rt}$
6. $d/dx\,[e^{g(x)}] = e^{g(x)}\,g'(x)$
7. $y - y_1 = m(x - x_1)$
8. $d/dx\,[\ln g(x)] = \frac{g'(x)}{g(x)}$
9. $P = Ae^{-rt}$