MATH221 CALCULUS I 100pts TEST 2 Spring'11 INSTRUCTOR: CARLAMORRIS

Page 1 NAME: Key

6pts 1. Is f (x) continuous at x = 2? (show work) $f(x)=\begin{cases}3x-5 & x=2\\ \dfrac{x^2-3x+2}{x-2} & x\neq 2\end{cases}$

$f(2)=3(2)-5=1$ exists ✓

$\lim_{x\to 2}\dfrac{(x-2)(x-1)}{x-2}=1$ exists ✓

$\boxed{\text{yes } f(2)=\lim_{x\to 2}\dfrac{x^2-3x+2}{x-2}}$

at $x=1$

$f(1)=\dfrac{1-3+2}{1-2}=0$

$\lim_{x\to 1} f(x)=0$

$0=0$

so cont at $x=1$

IN PROBLEMS 2-5 FIND THE INDICATED DERIVATIVES (5PTS EACH)

2. $f'(x)$ if $f(x) = 8x^5 + 3x^4 - 5x^3 + 9x + 1$

$\boxed{40x^4+12x^3-15x^2+9}$

3. $f'(t)$ if $f(t) = (6t^5 + 4t^3 + 3t)^{45}$

$\boxed{45(6t^5+4t^3+3t)^{44}(30t^4+12t^2+3)}$

4. $d/dt\ (5a^6t^5 + 7b^3t^3 + 7ct^2 - 2t + 3)$

$\boxed{25a^6t^4+21b^3t^2+14ct-2}$

5. $d^2/dr^2\ (8r^3 + 7r^2 + 9r + 6)\,\big|_{r=1}$

$24r^2+14r+9$

$(48r+14)\big|_{r=1}$

$\boxed{62}$

5pts 6. Find the equation of the tangent line to the curve $f(x) = 7x^2 + 5x + 7$ at $x = 2$

$f(2)=28+10+7=45$ pt $(2,45)$

$f'(x)=14x+5$

$f'(2)=33=m$

$\boxed{y-45=33(x-2)}$

8pts 7. Sketch the graph of a function that has the following properties, $f'(3) = 0$; $f(3) = 1$, $f(0) = 10$; concave up for all x.

see back p. 1

10pts 8. Locate all possible extrema of $f(x) = (1/3)x^3 + 4x^2 + 12x$. Also check for concavity and inflection points. Give intervals for increasing, decreasing, concavity, etc and then Sketch the graph.

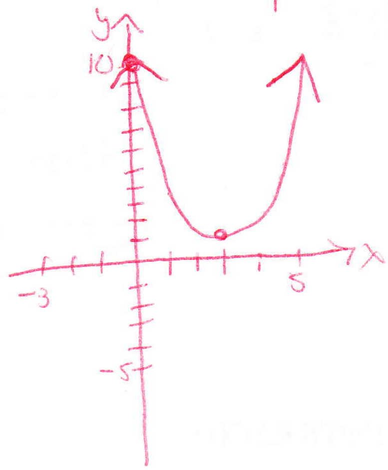
See back p. 1

(7) $f'(3)=0$ poss min max at $x=3$

$f(3)=1$ (3,1) a pt

$f(0)=0$ (0,10) a pt

Con up all x so (3,1) min



(8) $\frac{1}{3}x^3+4x^2+12x$

$f'(x)=x^2+8x+12$

$(x+2)(x+6)$

$(-7,\frac{8}{3})$

$(-8,-\frac{32}{3})$

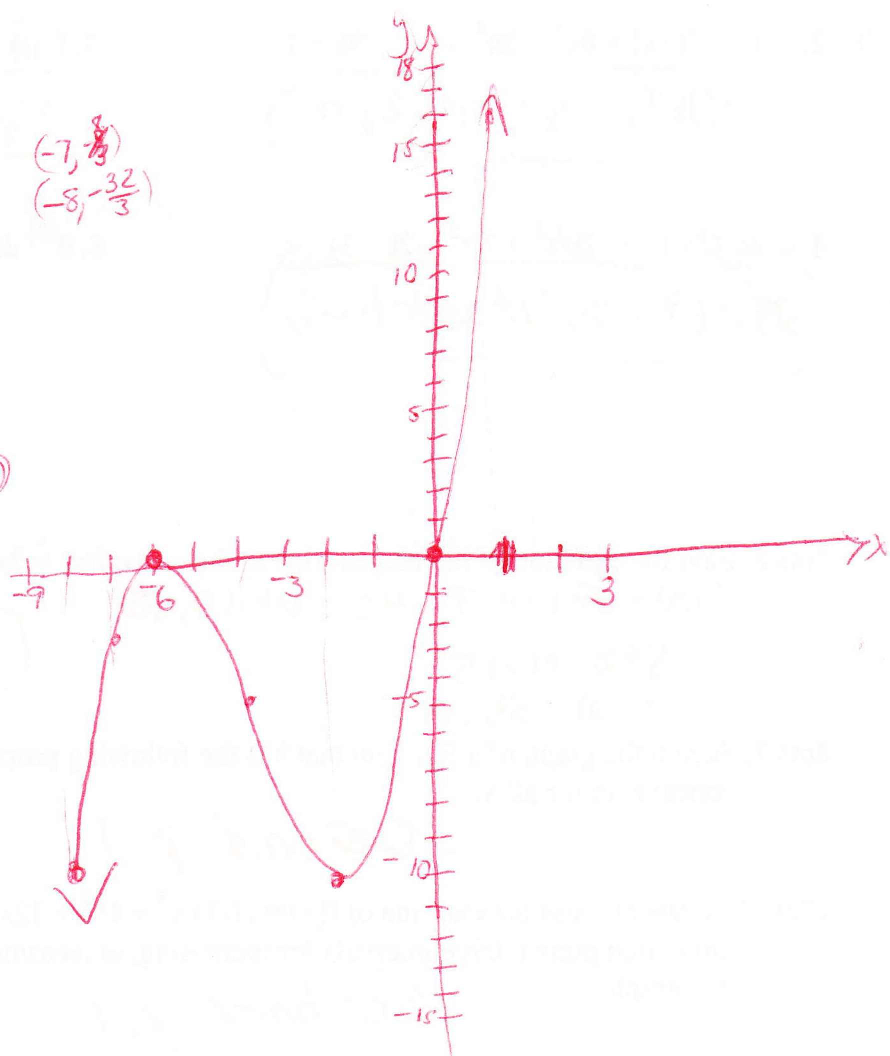
x+2	−	−	+
x+6	−	+	+
	⊕ −6	⊖ −2	⊕

inc $(-\infty,-6)\cup(-2,\infty)$

dec $(-6,-2)$

max $(-6,0)$

$-\frac{8}{3}+16-24$ min $(-2,-\frac{32}{3})$



$2x+8$

2x+8	−	+
	−4	

Con down $(-\infty,-4)$

$-\frac{64}{3}+64-48$ Up $(-4,\infty)$

inf pt $(-4,-\frac{16}{3})$

MATH221 CALCULUS I 100pts TEST 2 Spring'11 INSTRUCTOR: CARLA MORRIS

Page 2 NAME: Key

10pts 9. Graph $f(x) = x^4 - 2x^2$ by finding the x and y intercepts, relative extrema, inflection points, intervals increasing or decreasing and intervals of concavity.

See back page 2

8pts 10. Find the minimum value of $f(t) = 10t^3 - 15t^2 + 7$, $t > 0$ and give the value of t where this minimum occurs.

$30t^2 - 30t$

$30t(t-1)$

$0, 1$

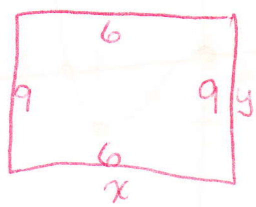
$30t$	$-$	$+$	$+$
$t-1$	$-$	$-$	$+$
	0 $\ominus$	1 $\oplus$	

min at $t=1$

$10-15+7$

$\boxed{\text{at } t=1 \text{ the min value is } 2}$

10pts 11. Suppose a man has \$720 build a rectangular enclosure. The north and south sides of the enclosure cost sides cost \$6 per running yard while the east and west sides which are more expensive more cost \$9 per running yard. Find the dimensions of the enclosure that will maximize the area of the enclosure.



$12x + 18y = 720$

max xy

$18y = 720 - 12x$

$y = 40 - \frac{2}{3}x$

$A(x) = x\left(40 - \frac{2}{3}x\right)$

$A(x) = 40x - \frac{2}{3}x^2$

$A'(x) = 40 - \frac{4}{3}x$

$40 - \frac{4}{3}x$ $\quad + \mid -$ at 30

max

$x = 30$

$y = 20$

$\boxed{\text{Max area is } 30\text{yd} \times 20\text{yd or } 600\text{yd}^2}$

$x^4 - 2x^2$

$x^2(x^2-2)$

$x = 0, \pm\sqrt{2}$ x interc

$f'(x) = 4x^3 - 4x$

$4x(x-1)(x+1)$

$4x$	$-$	$-$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$	$+$
	$\ominus$	$\oplus$	$\ominus$	$\oplus$

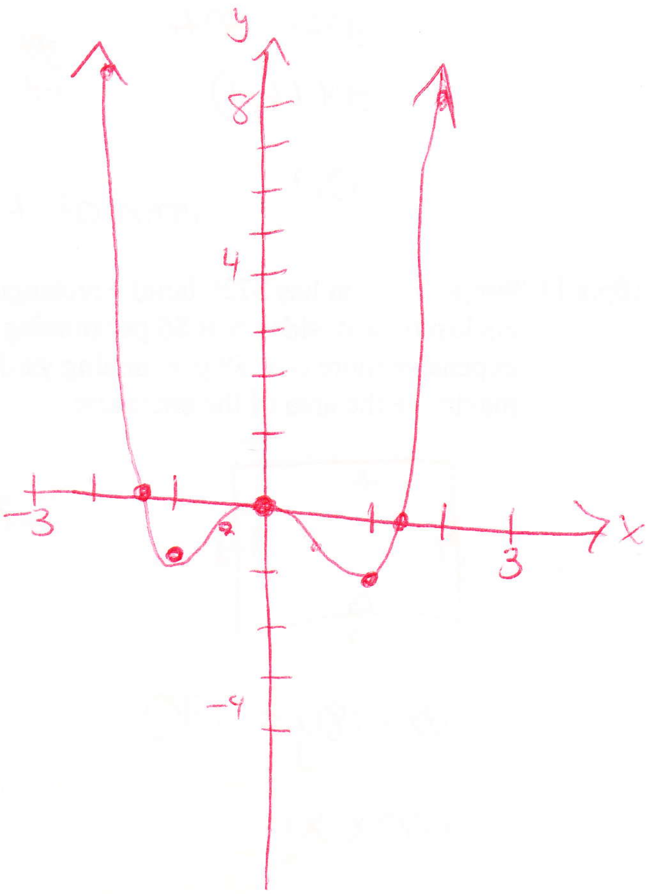
$-1 \quad 0 \quad 1$

dec $(-\infty,-1) \cup (0,1)$

inc $(-1,0) \cup (1,\infty)$

local min $(\pm 1, -1)$

local max $(0,0)$



$f''(x) = 12x^2 - 4$

$x^2 = 1/3$

$x = \pm\sqrt{3}/3$

$12x^2-4$

$\oplus \quad \ominus \quad \oplus$

$-\frac{\sqrt{3}}{3} \quad \frac{\sqrt{3}}{3}$

Con up $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$

Con down $(-\sqrt{3}/3, \sqrt{3}/3)$

Infpt $(\pm\sqrt{3}/3, -5/9)$

$\frac{1}{9} - \frac{2}{3}$

NAME: Key

8pts 12. Given the cost function $C(x) = x^3 - 15x^2 + 100x + 150$ find the minimum marginal cost.

$C'(x) = 3x^2 - 30x + 100 = MC$

$C''(x) = MC' = 6x - 30$

$6x - 30$

$- \quad +$

5 min

$75 - 150 + 100$

When $x = 5$
min MC is \$25

15pts 13. Suppose the consumer demand for a certain item as a function of its price **p** is given by **x** = D (**p**) = 40 – (p / 6). Determine the production level and price that maximizes the profit if the cost output function is given by

$C(x) = x^3 + 9x^2 - 360x + 2{,}000$

$0 \le x \le 40$

$0 \le p \le 240$

$6x = 240 - p$

$p = 240 - 6x$

Rev $240x - 6x^2$

$Pr(0) = -2000$

$Pr(40) = -66{,}000$

$Pr(10) = 1500$

$Pr(x) = (240x - 6x^2) - (x^3 + 9x^2 - 360x + 2000)$

$Pr(x) = -x^3 - 15x^2 + 600x - 2000$

$Pr'(x) = -3x^2 - 30x + 600$

$-3(x^2 + 10x - 200)$

$-3(x + 20)(x - 10)$

$x = 10$ or $\cancel{x = -20}$

Max profit \$1500
when produce 10 units
at \$180 each

formulas

$y - y_1 = m(x - x_1)$ $\quad ax + by = c \quad y = mx + b \quad m_1 = m_2$

$m_1 = -1/m_2 \quad m = \dfrac{y_2 - y_1}{x_2 - x_1} \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$