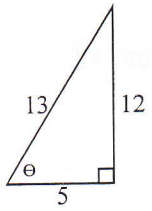
MATH117 Pre-Calculus for Scientists and Engineers **SAMPLE TEST 4 (page 1)**

4pts 1. a) Convert $\frac{-5\pi}{4}$ radians to degrees. $-\frac{5\pi}{4}\cdot\left(\frac{180^\circ}{\pi}\right) = -225^\circ$

b) Convert 96° to radians $96^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{8\pi}{15}$

4pts 2. Find the length of an arc that subtends a central angle of 2radians in a circle with radius 2 miles. $s = r\theta = 2(2) =$ 4miles

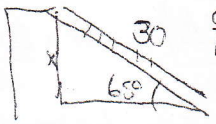
4pts. 3.



Given the right triangle to the left determine a) $\sin\theta$ $O/H = 12/13$

b) $\tan\theta$ $O/A = 12/5$

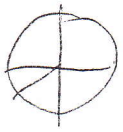
4pts. 4. A thirty foot ladder leans against a building so that the angle between the ground an the ladder is 65°. How high does the ladder reach on the building?



O/H $\sin 65^\circ = x/30$ $x = 30\sin 65^\circ$ $x \approx 27.2$ ft

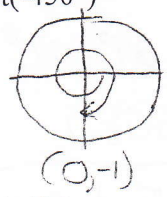
In problems 5-8 use a unit circle, give the reference angle and quadrant, and then use trigonometric definition to give the numerical answer. (4 points each)

5. $\sin(210^\circ)$ $-.5$



30° ref QIII $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ y value

6. $\cot(-450^\circ)$



$x/y = 0/-1 = 0$ $(0,-1)$

7. $\cos(300^\circ)$ $\frac{1}{2}$

ref 60° QIV $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ x value

8. $\sec(210^\circ)$ $-2\sqrt{3}/3$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\frac{1}{x}$

4pts 9. Given $\tan\theta = -5/12$ and $\cos\theta > 0$ a) Find $\sin\theta$ $-5/13$

QIV $y = -5/13$ y neg $x = 12/13$

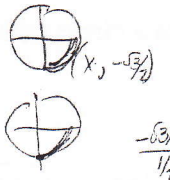
b) Find $\cot\theta$ $-12/5$

4pts 10. Find the area of a triangle with sides of length 10cm and 2cm with included angle 120°.

$\frac{1}{2}ab\sin\theta$ $\frac{1}{2}(10)(2)(\sin 120^\circ) = 10\sin 120^\circ = 10\left(\frac{\sqrt{3}}{2}\right) = \boxed{5\sqrt{3}}$

$\approx 8.66\,cm^2$

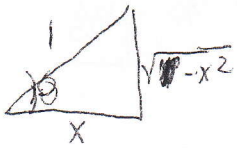
4pts 11. Find the exact value of a) $\sin^{-1}\frac{-\sqrt{3}}{2} = \boxed{-\pi/3}$



b) $\tan^{-1}(-\sqrt{3})$ $\boxed{-\pi/3}$

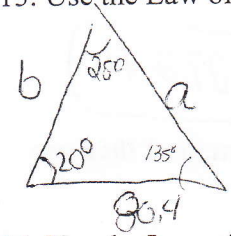
4pts 12. Rewrite the expression as an algebraic expression in x: $\sin(\cos^{-1}x)$

$\frac{A}{H} = \frac{X}{1}$



$\frac{O}{H} = \frac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$

4pts 13. Use the Law of Sines to solve the triangle ABC: c = 80.4, ∠A = 20°, ∠C = 25°



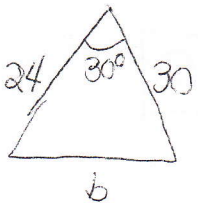
$\angle B = 180^\circ - 20^\circ - 25^\circ = \boxed{135^\circ = \angle B}$

$\frac{\sin 25^\circ}{80.4} = \frac{\sin 135^\circ}{b} = \frac{\sin 20^\circ}{a}$

$a = \frac{80.4\sin 20^\circ}{\sin 25^\circ}$ $\boxed{a \approx 65.1}$

$b = \frac{80.4\sin 135^\circ}{\sin 25^\circ}$ $\boxed{b \approx 134.5}$

4pts 14. Use the Law of Cosines to determine side b: a = 24, c = 30, ∠B = 30°



$b^2 = 24^2 + 30^2 - 2(24)(30)\cos 30^\circ$

$b^2 = 228.7$

$\boxed{b = 15.1}$

4pts 15. Simplify the trigonometric expression $\frac{\sec^2 x - 1}{\sec^2 x}$

$1 + \tan^2 x = \sec^2 x$

$\sec^2 x - 1 = \tan^2 x$

$\frac{\tan^2 x}{\frac{1}{\cos^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \boxed{\sin^2 x}$

4pts 16. Verify $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

$\dfrac{\sec^2 x}{1-\frac{\sin^2 x}{\cos^2 x}} = \dfrac{\sec^2 x \cos^2 x}{\cos^2 x - \sin^2 x} = \dfrac{\frac{1}{\cos^2 x}(\cos^2 x)}{\cos^2 x - \sin^2 x} = \boxed{\dfrac{1}{\cos^2 x - \sin^2 x}}$

$\pi/4 = 3\pi/12$
$\pi/6 = 2\pi/12$
$\pi/3 = 4\pi/12$
$\pi/2 = 6\pi/12$

4pts 17. Find the exact value of $\cos\left(\dfrac{13\pi}{12}\right)$ using an addition or subtraction formula.

$\cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \cos 3\pi/4 \cos \pi/3 - \sin 3\pi/4 \sin \pi/3$

$\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\dfrac{-\sqrt{2}-\sqrt{6}}{4}}$

4pts. 18. Prove the cofunction identity below using and addition or subtraction formula

$\tan\left(\dfrac{\pi}{2} - u\right) = \cot u$

$\dfrac{\tan \pi/2 + \tan u}{1 + \tan \pi/2 \tan u}$ so use $\dfrac{\sin(\pi/2 - u)}{\cos(\pi/2 - u)} = \dfrac{\sin \pi/2 \cos u - \cos \pi/2 \sin u}{\cos \pi/2 \cos u + \sin \pi/2 \sin u}$

~~$\tan \pi/2$ undef~~

$\dfrac{(1)\cos u - (0)\sin u}{(0)\cos u + (1)\sin u} = \dfrac{\cos u}{\sin u} = \boxed{\cot u}$

4pts 19. Verify $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$\boxed{2\sin\theta\cos\theta}$

4pts 20. Use an appropriate half-angle formula to find the exact value of sin22.5°.

$\sin 22.5^\circ$ positive $\sin 22.5^\circ = \pm\sqrt{\dfrac{1-\cos 45^\circ}{2}} = \pm\sqrt{\dfrac{1-\frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\dfrac{2-\sqrt{2}}{4}}$

$\boxed{\frac{1}{2}\sqrt{2-\sqrt{2}}}$

4pts 21. Write the product as a sum sin2xsin5x.

$\cos(-\theta) = \cos\theta$

$\frac{1}{2}\left[\cos(2x-5x) - \cos(2x+5x)\right] = \frac{1}{2}\left[\cos(-3x) - \cos 7x\right]$

$\boxed{\frac{1}{2}\cos 3x - \frac{1}{2}\cos 7x}$

4pts 22. Verify $\dfrac{\sin 4x}{\sin x} = 4\cos x \cos 2x$

$\sin 4x = 2\sin 2x \cos 2x$

$\dfrac{2\sin 2x \cos 2x}{\sin x}$

$2\left[\dfrac{2\sin x \cos x}{\sin x}\right]\cos 2x = \boxed{4\cos x \cos 2x}$

4pts 23. Find all solutions on $[-2\pi, 2\pi]$ where $\cos = \frac{-1}{2}$

$\pi/3$, ref
QII, III

$-2\pi/3, -4\pi/3, 2\pi/3, 4\pi/3$

4pts 24. Find all solutions on $[0, 2\pi]$ where $3\sin^2\theta - 7\sin\theta + 2 = 0$

$(3\sin\theta - 1)(\sin\theta - 2) = 0$

.34 radians

$\sin\theta = 1/3$ or ~~$\sin\theta = 2$~~

.34 radians and 2.80 radians

and
3.14
− .34
2.80

4pts 25. Find all solutions on $[-2\pi, 2\pi]$ where $2\sin^2\theta + \cos\theta = 1$

$2(1 - \cos^2\theta) + \cos\theta - 1 = 0$

$1 - 2\cos^2\theta + \cos\theta = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -1/2$

$\cos\theta = 1$

$2\pi/3, 4\pi/3, -2\pi/3, -4\pi/3$
$0, 2\pi, -2\pi$

4pts 26. Find all solutions on $[0, 2\pi]$ where $\cos\frac{\theta}{2} - 1 = 0$

$\cos\frac{\theta}{2} = 1$

0

$\frac{\theta}{2} = 0, 2\pi$ $\theta = 0, 4\pi$ etc

4pts 27. Find all solutions on $[0, 2\pi]$ where $\cos 2\theta + \cos\theta = 2$

$(\cos^2\theta - \sin^2\theta) + \cos\theta = 2$

$(2\cos^2\theta - 1) + \cos\theta = 2$

$2\cos^2\theta + \cos\theta - 3 = 0$

$(2\cos\theta + 3)(\cos\theta - 1) = 0$

~~$\cos\theta = -3/2$~~ $\cos\theta = 1$

$0, 2\pi$