MATH117 Pre-Calculus for Scientists and Engineers **SAMPLE TEST 3 (page 1)**

1. Sketch the graph of $y = 2^x - 3$ see graph paper

2. Sketch the graph of $y = e^{x+1}$ See graph paper

3. When a certain medical drug is administered to a patient, the number of milligrams remaining in the patient's bloodstream after t hours is modeled by $D(t) = 50e^{-0.2t}$
   How many milligrams of the drug remain in the patient's bloodstream after 3 hours?

   $50e^{-.2(3)} = 50e^{-.6} =$ 27.44 mg

4. If $5000 is invested at an interest rate of 4 % per year and interest is compounded continuously how much is it worth after 18 years?

   $5000\,e^{.04(18)}$

   $5000\,e^{.72} =$ $10,272.17

5. a) Express in exponential form $\log_5 625 = 4$ $5^4 = 625$

   b) Express in logarithmic form $2^{-3} = \frac{1}{8}$ $\log_2 \frac{1}{8} = -3$

6. a) Evaluate $\log_7 49$ $7^x = 49$ $7^x = 7^2$ $x = 2$

   b) Evaluate $\ln e^7$ $7 \ln e$ 7

7. Graph $y = \log_4 x$ see graph paper

8. Write as a single logarithm: $3\ln x + 4\ln y + 4\ln(z + 1)$

   $\ln x^3 + \ln y^4 + \ln(z+1)^4$ $\ln x^3 y^4 (z+1)^4$

9. Use the laws of logarithms to expand $\log_3 \left(\frac{x^3(x+1)}{(x-5)^2} \right)$

   $\log_3 x^3 + \log_3(x+1) - \log_3(x-5)^2$

   $3\log_3 x + \log_3(x+1) - 2\log_3(x-5)$

MATH117 Pre-Calculus for Scientists and Engineers **SAMPLE TEST 3 (page 2)**

4pts 10. Solve $e^{2x} - 4e^x + 3 = 0$

$(e^x-3)(e^x-1)=0$ $e^x=3$ or $e^x=1$ $x=0$ or $\ln 3$

4pts 11. Solve $\log_4 x + \log_4(3x - 8) = 2$

$\log_4(x)(3x-8)=2$

$3x^2-8x=16$

$3x^2-8x-16=0$

$(3x+4)(x-4)=0$

$x=-4/3, 4$ $x=4$

4pts 12. Solve $\log_2(x - 3) + \log_2(x - 2) < 1$

Domain: $x>2$, $x>3$, $x>3$

$\log_2(x-3)(x-2)<1$

$(x-3)(x-2)<2^1$

$x^2-5x+6<2$

$x^2-5x+4<0$

$(x-4)(x-1)<0$

$x-1$	−	+		+
$x-4$	−	−		+
	+	⊖	+	
	1		4	

$[3,4]$

4pts 13. A fox population in a certain region has a relative growth rate of 8% per year. It is estimated the population in 2005 was 18,000. Find a function $n(t) = n_0e^{rt}$ that models the population t years after 2005.

$n(t) = 18{,}000e^{.08t}$

4pts 14. A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling so its temperature at time t is $T(t) = 70 + 130e^{-0.04855t}$ where t is measured in minutes and T is the Fahrenheit temperature.

a) What is the initial temperature of the soup? $70+130$ $200°F$

b) What is the soup temperature after 15 minutes?

$70 + 130e^{-.04855(15)}$

about $132.76°F$

4pts 15. First determine the value of x and then determine the value of the tangent at P. The point P is in QII of the unit circle.

P(x, 3/5)

x neg

$x^2 + (3/5)^2 = 1$

$x^2 = 16/25$

$x = \pm 4/5$

$x = -4/5$

$y/x = \frac{3/5}{-4/5} = -3/4$ tangent

In problems 16 – 21 use a unit circle, give the reference angle and quadrant, and then use trigonometric definition to give the numerical answer. (4 points each)

16. $\sin(10\pi)$ (1,0) y value 0

17. $\tan\left(-\frac{3\pi}{2}\right)$ (0,1) undef $\frac{1}{0}$

18. $\cos\left(\frac{5\pi}{6}\right)$ $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $-\frac{\sqrt{3}}{2}$

19. $\sec\left(\frac{5\pi}{3}\right)$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\frac{1}{x}$ 2

20. $\csc\left(\frac{17\pi}{4}\right)$ $\sqrt{2}$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$ $\frac{1}{y}$

21. $\cot\left(\frac{-5\pi}{6}\right)$ $-\sqrt{3}$ $\frac{x}{y}$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

4pts 22. Graph $y = 25\sin(4x)$ see graph paper

4pts 23. Graph $y = -10\cos\left(2x - \frac{\pi}{3}\right)$ see graph paper

4pts 24. Graph $y = 5\sec\left(\frac{1}{2}x\right)$ see graph paper

4pts 25. Evaluate a) $\cos^{-1}\left(\cos\frac{5\pi}{6}\right)$ $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 5\pi/6$

b) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4$

4pts 26. Evaluate $\tan\left(\tan^{-1}\frac{4\pi}{3}\right)$ $4\pi/3$

4pts. 27. Find a function that models the simple harmonic motion having the given properties. Assume displacement is zero is at time t = 0. $35\sin \pi/4\, t$

Amplitude 35cm period 8s

$8 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{8}$

Some formulas you may need

($y = a\sin\omega t$ amplitude $|a|$ period $\frac{2\pi}{\omega}$ frequency $\frac{\omega}{2\pi}$

Asin(Bx + C) or Acos(Bx + C), |A| = amp., period = 2π/ B, p.s. = -C / B

$T(t) = T_s + D_0 e^{-kt}$ D_0 is initial difference in temperature between an object and its surroundings

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\left(\frac{-b}{2a}, \frac{-b^2 + 4ac}{4a}\right)$ $A = Pe^{rt}$ $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$\cos^2\theta + \sin^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

MATH 117 Precalc for Scientists & Engineers
Sample Test 3

① $y = 2^x - 3$

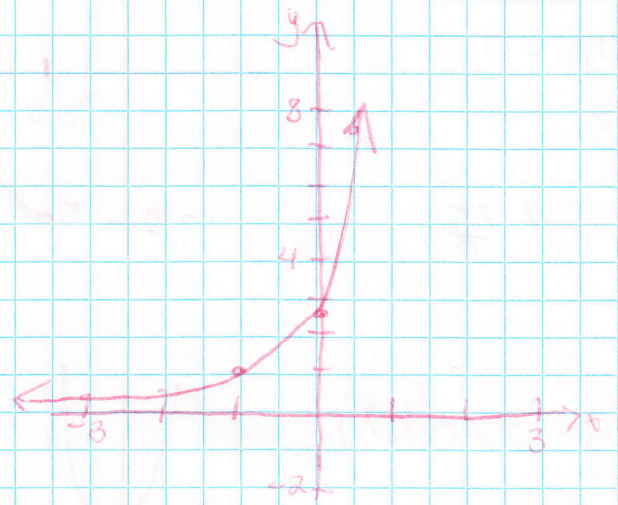
x	y
-2	-2.75
-1	-2.5
0	-3
1	-2
2	1
3	5

Horiz asym $y = -3$

② $y = e^{x+1}$

x	y
-2	.37
-1	1
0	2.7
1	7.4
-3	.14

asym x a x is



③ $y = \log_4 x$

$4^y = x$

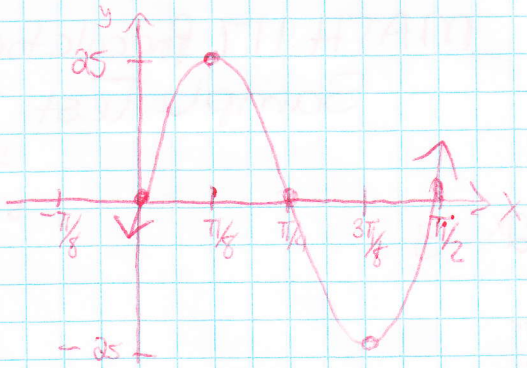
y	x
-2	1/16
-1	1/4
0	1
1	4
2	16

(22) $y = 25\sin 4x$

Amp 25

period $2\pi/4 = \pi/2$

mark $\pi/8$



(23) $y = -10\cos(2x - \pi/3)$

Amp 10

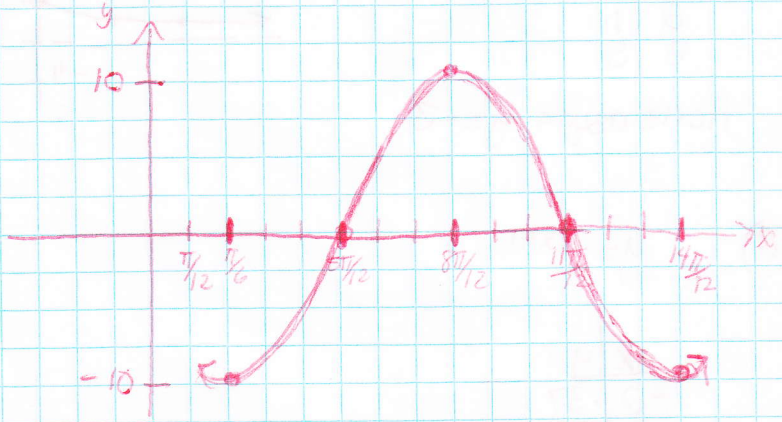
period $2\pi/2 = \pi$

ps $2x = \pi/3$

$x = \pi/6$

Start $\pi/6$ end $7\pi/6$

$\pi/4$



(24) Graph $y = 5\sec(\frac{1}{2}x)$

recip cosine

period 4π

~~max~~ 0, ~~2π~~, 4π

~~min~~ 2π ~~max~~

Zeroes (undef) π, 3π