MATH117 Pre-Calculus for Scientists and Engineers **SAMPLE TEST 2 (page 1)**

1. Given $f(x) = x^2 + 7x + 6$ find $\dfrac{f(a+h) - f(a)}{h}$

$\dfrac{[(a+h)^2 + 7(a+h) + 6] - [a^2 + 7a + 6]}{h}$

$\dfrac{a^2 + 2ah + h^2 + 7a + 7h + 6 - a^2 - 7a - 6}{h}$

$\dfrac{h(2a+h+7)}{h}$

$2a+h+7$

2. Find the domain of the function $\dfrac{2x+7}{x^3 - 16x}$

$\dfrac{2x+7}{x(x-4)(x+4)}$

D: $x \neq 0, \pm 4$

3. Graph $f(x) = \begin{cases} x^2 + 2 & x \leq -2 \\ 3x + 2 & x > -2 \end{cases}$ see other page

4. Graph $f(x) = x^3 + 1$ See other page

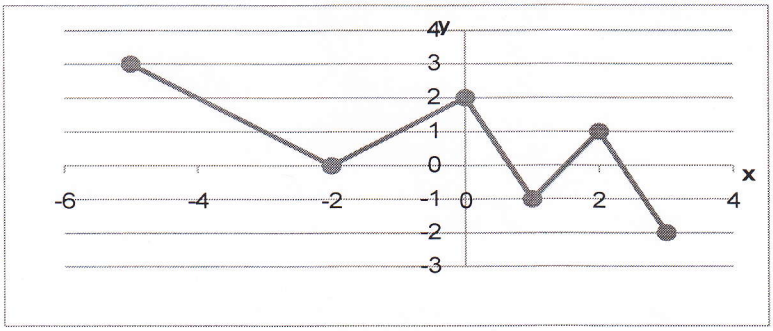
5. Determine whether the equation defines y as a function of x: $x^2 + y^2 = 4$

$y^2 = \sqrt{4 - x^2}$

$y = \pm\sqrt{4 - x^2}$

No since each x value gives 2 y values

6. The graph of a function is given below. Determine the intervals of which the function
a)increases and b) decreases



inc $(-2,0) \cup (1,2)$

dec $(-5,-2) \cup (0,1) \cup (2,3)$

7. Determine the average rate of change $\dfrac{f(b) - f(a)}{b - a}$ of the function between the given values of the variable $f(x) = x^2 - 4x$; $x = 1$ and $x = 3$

$\dfrac{f(3) - f(1)}{3-1} = \dfrac{[9-12] - [1-4]}{2} = \dfrac{-3 - (-3)}{2} = 0$

8. Graph $f(x)=|x|-3$ See other page

9. Graph $f(x)=(x+1)^2-4$ See other page

10. Let $f(x)=x^2+3x+5$ and $g(x)=x^3+7x^2+9$ find $(f-g)(x)$

$(x^2+3x+5)-(x^3+7x^2+9) = -x^3-6x^2+3x-4$

11. Let $f(x)=\dfrac{x}{3x+5}$ and $g(x)=2x-3$ find $(gof)(x)$

$g\left(\frac{x}{3x+5}\right) = 2\left(\frac{x}{3x+5}\right)-3$

12. Express the function h(x) as a composite of two functions f(x) and g(x) so h(x) = (fog)(x))

$h(x)=\sqrt[3]{x^2+5x+7}$

$f(x)=\sqrt[3]{x}$

$g(x)=x^2+5x+7$

13. Find $f^{-1}(x)$ when $f(x)=\dfrac{2x+5}{x-7}$

$y=\frac{2x+5}{x-7}$ $\quad xy-7x=2y+5$

$x=\frac{2y+5}{y-7}$ $\quad xy-2y=7x+5$

$(x-2)y=7x+5$

$y=\frac{7x+5}{x-2}=f^{-1}(x)$

14. Explain whether the graph of $f(x)=|x|-3$ (problem 8 above) is one-to-one.

It is not one to one. It is a fnc. (passes VLT) but it fails horizontal line test.

15. Explain whether the quadratic has a maximum or minimum and then find that value if $f(x)=2x^2+7x-15$

$a=2$ $\quad x=-\frac{7}{4}$

$a>0$

min

$2\left(-\frac{7}{4}\right)^2+7\left(-\frac{7}{4}\right)-15$

$\frac{49}{8}-\frac{49}{4}-15$

$\frac{49}{8}-\frac{98}{8}-\frac{120}{8}$

$-\frac{169}{8}$

min value $-169/8$ at $x=-7/4$

16. Express $f(x)=2x^2+7x-15$ in the form $a(x-h)^2+k$

$2(x^2+\frac{7}{2}x)-15$

$2(x^2+\frac{7}{2}x+\frac{49}{16})-15-\frac{49}{8}$

$2(x+\frac{7}{4})^2-\frac{169}{8}$

4pts 17. A manufacturer finds that the revenue generate by selling x units of a certain commodity is given by the function $R(x) = 120x - 10x^2$ where R(x) is measured in dollars. What is the maximum revenue and how many units should be manufactured to obtain this maximum?

$x = \frac{-120}{2(-10)} = 6$

Produce 6 units for max revenue of $360

4pts 18. $P(x) = x^3 - 4x$ Factor the polynomial and use the factored form to find the zeroes. Sketch the graph using x intercepts, what the graph looks like near each x intercept and end behavior to do the sketch

$x(x-2)(x+2)$

zeroes at $0, \pm 2$

near 0 $(x)(0-2)(0+2)$ $-4x$ \

near 2 $(2)(x-2)(2+2)$ $8(x-2)$ /

near -2 $(-2)(-2-2)(x+2)$ $8(x+2)$ /

1	-3
3	15
-1	3
-3	-15

4pts 19. Use long division to find the quotient and remainder $\frac{x^3 - x^2 - 2x + 6}{x - 2}$

$x-2$) $x^2 + x$

$x^3 - x^2 - 2x + 6$

$x^3 - 2x^2$

$x^2 - 2x + 6$

$-(x^2 - 2x)$

6

Quotient $x^2 + x$

Remainder 6

4pts 20. Find a polynomial of degree four with zeroes -2, -1, 2, and 4.

$[x-(-2)][x-(-1)][x-2][x-4] = (x+2)(x+1)(x-2)(x-4)$

$(x^2+3x+2)(x^2-6x+8) = x^4 - 3x^3 - 8x^2 + 12x + 16$

4pts 21. If $P(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$ determine all rational zeroes and then completely factor the Polynomial.

possible

Possible $\pm 1, \pm 2, \pm 4, \pm 8$

$P(1) = 0$

$x-1$ is a factor

$x-1$) $x^3 + 7x^2 + 14x + 8$

$x^4 + 6x^3 + 7x^2 - 6x - 8$

$x^4 - x^3$

$7x^3 + 7x^2 - 6x - 8$

$-(7x^3 - 7x^2)$

$14x^2 - 6x - 8$

$14x^2 - 14x$

$8x - 8$

$8x - 8$

0

$P(-2) = 0$ so $x-(-2)$ or $x+2$ is a factor

$x+2$) $x^2 + 5x + 4$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 2x^2$

$5x^2 + 14x + 8$

$5x^2 + 10x$

$4x + 8$

$4x + 8$

0

$x^2 + 5x + 4 = (x+1)(x+4)$

$(x-1)(x+1)(x-2)(x+4) = P(x)$

4pts 22. Perform the indicated operation below and write the result in the form a + bi

$(3+5i)(4-2i)$

$12 - 10i^2 + 20i - 6i$

$22 + 14i$

4pts 23. Find all solutions and express in the form a + bi $4x^2 - 24x + 37 = 0$

$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(4)(37)}}{2(4)}$

$\frac{24 \pm \sqrt{-16}}{8} = \frac{24}{8} \pm \frac{4i}{8} = 3 \pm \frac{1}{2}i$

4pts 24. Factor $x^3 - 3x^2 + x - 3$ completely

$\pm 1, \pm 3$

$P(3) = 0$

So $x-3$ is a factor

either use long division or try grouping

$x^2(x-3) + 1(x-3)$

$(x^2+1)(x-3)$

$(x-i)(x+i)(x-3)$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm i$

4pts 25. Find a polynomial with integer coefficients that has degree 5 and a zero at 0, and zeroes at $\pm 2i$ both with a multiplicity of 2.

$(x)(x-2i)^2(x+2i)^2$

$(x)[(x-2i)(x+2i)]^2$

$x[x^2+4]^2$

$x(x^4 + 8x^2 + 16)$

$x^5 + 8x^3 + 16x$

4pts. 26. Find all vertical and horizontal asymptotes for $y = \frac{x^2 - 1}{x^2 - 5x + 6}$

$\frac{(x-1)(x+1)}{(x-2)(x-3)}$

vert asym $x = 2$ or 3

horiz $y = 1$

4pts 27. Graph $f(x) = \frac{x^2}{x^2 - 4}$ See other page

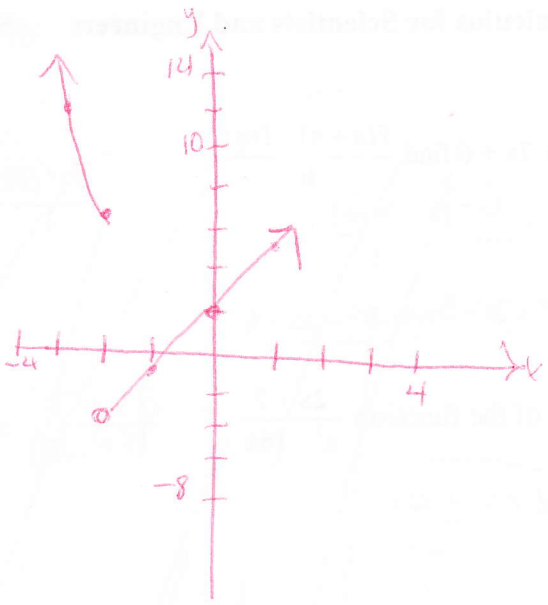
Some formulas you may need

1. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

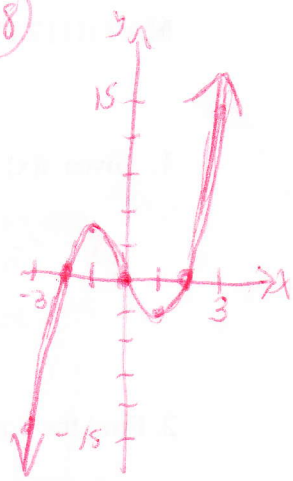
2. $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

(3) $f(x) = x^2 + 2 \quad x \leq -2$

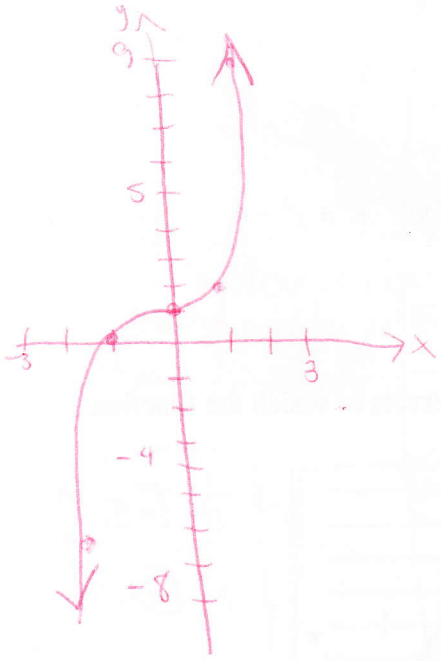
$3x + 2 \quad x > -2$



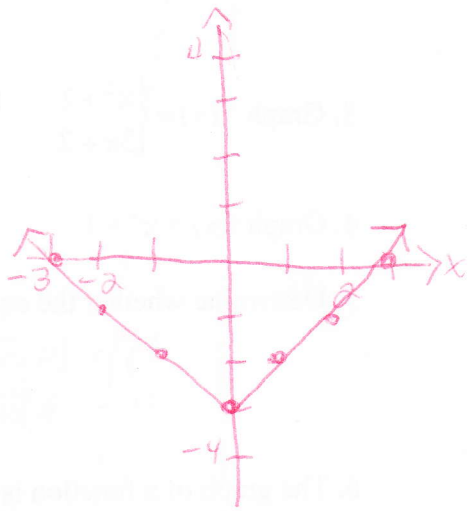
(18)



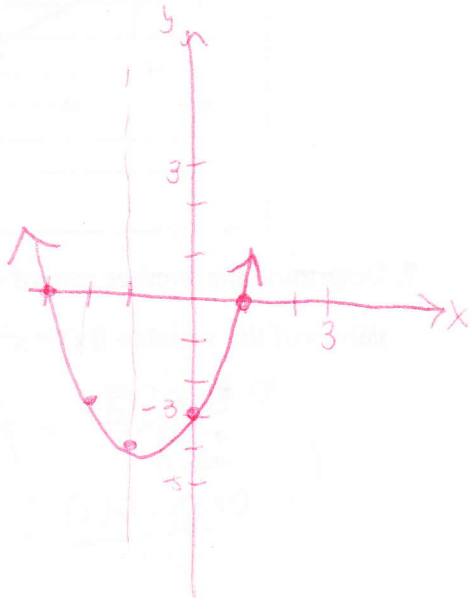
(4) $f(x) = x^3 + 1$



(8) $f(x) = |x| - 3$



(9) $f(x) = (x+1)^2 - 4$



(#3) $\frac{x^2}{x^2-4}$

Vert asym $x = \pm 2$

horiz $y = 1$

Cross horiz NG

$1 = \frac{x^2}{x^2-4}$

Symm about y