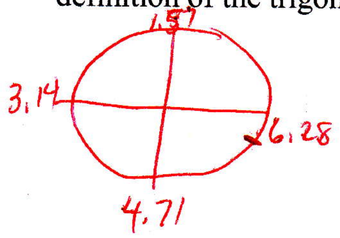
MATH115 PRECALCULUS 100pts TEST 4 FALL 2011 INSTRUCTOR: CARLA MORRIS
p.1 NAME: Key

4pts 1. Convert $\frac{7\pi}{6}$ radians to degrees.

$\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = \frac{7(180^\circ)}{6} = \boxed{210^\circ}$

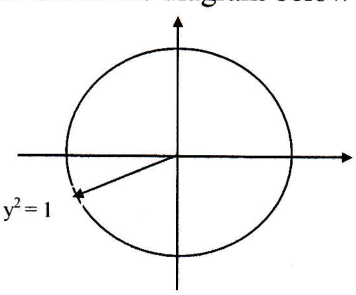
4pts 2. Would sin6 be positive or negative? Use unit circle to determine the quadrant and use definition of the trigonometric function in order to answer the question.



Q IV
y negative
sine is y

$\boxed{\text{Negative}}$

4pts 3. Given the diagram below and that x = -15/17 find cosθ and tanθ.


$x^2 + y^2 = 1$

$\left(-\frac{15}{17}\right)^2 + \sin^2\theta = 1$

$\frac{225}{289} + \sin^2\theta = 1$

$\sin^2\theta = \frac{64}{289}$

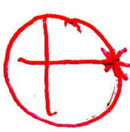
$\boxed{\cos\theta = -15/17}$ ✓

$\sin\theta = \boxed{-8/17}$

$\boxed{\tan\theta = \frac{-8}{-15} = \boxed{8/15}}$ ✓

In problems 4-11 use a unit circle, give the reference angle and quadrant, and then use trigonometric definition to give the numerical answer. (4 points each)

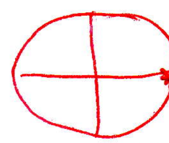
4. tan(2 π) y/x


(1,0) 0/1 $\boxed{0}$

5. sin(3π/2) y value


(0,-1) $\boxed{-1}$

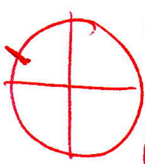
6. csc(360°)


(1,0)

1/y

1/0 $\boxed{\text{undefined}}$

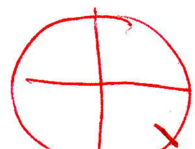
7. sin(150°)


ref 30°
Q II
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
y value
$\boxed{\frac{1}{2}}$

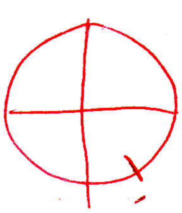
8. tan(240°) $\boxed{\sqrt{3}}$

60° ref
Q III
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$
y/x

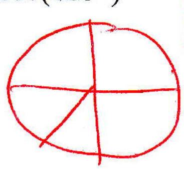
9. cos(7π/4)


ref π/4
Q IV
$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
x value
$\boxed{\frac{\sqrt{2}}{2}}$

10. cot(5π/3) $\boxed{-\frac{\sqrt{3}}{3}}$


ref π/3
Q IV
$(+\frac{1}{2}, -\frac{\sqrt{3}}{2})$
x/y

11. sec(225°) $\boxed{-\sqrt{2}}$


ref 45°
Q III
$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
1/x

MATH115 PRECALCULUS 100pts TEST 4 FALL 2011 INSTRUCTOR: C.MORRIS

p.2 NAME: Key

4pts 12. Prove the following is an identity $\tan\theta\,(\tan\theta + \cot\theta) = \sec^2\theta$

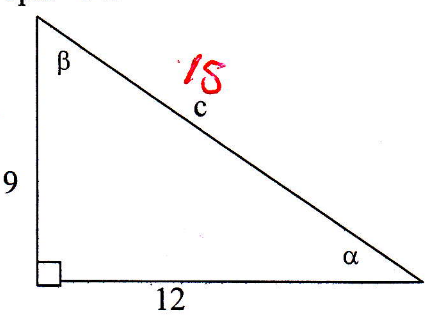
$\tan^2\theta + 1$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \boxed{\sec^2\theta}$

4pts 13. Simplify $\dfrac{\sin\theta + \cos\theta}{\cot\theta + 1}$

$\dfrac{(\sin\theta+\cos\theta)}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\sin\theta}} = \dfrac{\sin\theta+\cos\theta}{\frac{\cos\theta+\sin\theta}{\sin\theta}} = \boxed{\sin\theta}$

6pts 14.



9^2+12^2

$81+144$

$225 = c^2$

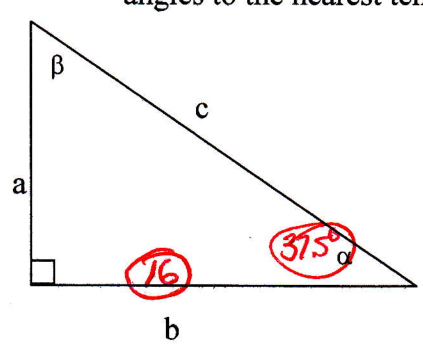
using the right triangle to the left first find the length c then find the following:

a) $\sin\alpha$ O/H $\boxed{9/15}$

b) $\cos\beta$ A/H $\boxed{9/15}$

c) $\tan\alpha$ O/A $\boxed{9/12}$

6pts 15. Given $\alpha = 37.5°$, $b = 16$ solve the triangle round lengths to two decimal places and angles to the nearest tenth of a degree.



$\boxed{\beta = 90° - 37.5° = 52.5°}$

$\tan\alpha = \frac{a}{16}$

$\tan 37.5° \cdot 16 = a$

$\boxed{12.28 \approx a}$

$\cos 37.5° = \frac{16}{c}$

$c = \frac{16}{\cos 37.5°} = 20.17$

$\boxed{c = 20.17}$

4pts 16. Prove the following is an identity $\dfrac{1 - \cot^2\theta}{1 + \cot^2\theta} = 2\sin^2\theta - 1$

$\dfrac{1 - \frac{\cos^2\theta}{\sin^2\theta}}{\csc^2\theta}$

$\dfrac{\sin^2\theta - \cos^2\theta}{\sin^2\theta} \cdot \sin^2\theta$

$\frac{1}{\csc^2\theta} = \sin^2\theta$

$\dfrac{\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta}}{\csc^2\theta}$

$\sin^2\theta - \cos^2\theta$

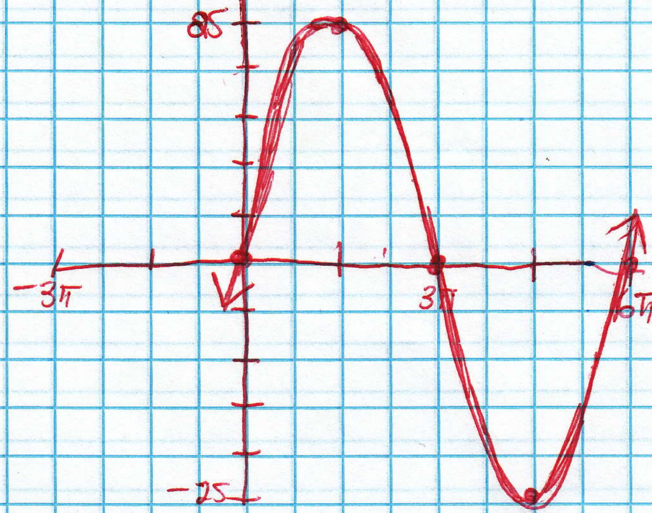
$\sin^2\theta - (1 - \sin^2\theta) = \boxed{2\sin^2\theta - 1}$

Test 4 Math 115 Fall 2011

5pts

18 $y = 25 \sin \frac{1}{3}x$

Amp 25
period 6π
mark $3\pi/2$
PS None

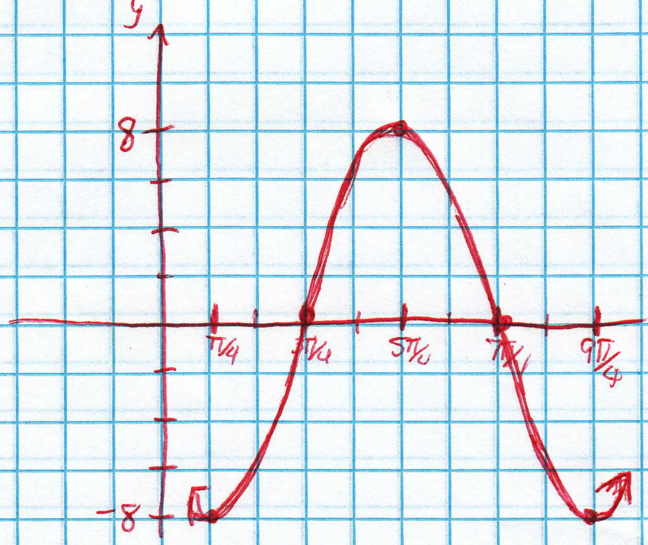


5pts

19 $y = -8\cos(x - \pi/4)$

Amp 8
period 2π
P.S. $\pi/4$
Start $\pi/4$
end $9\pi/4$

Mark $\pi/2$



MATH115 PRECALCULUS 100pts TEST 4 FALL 2011 INSTRUCTOR: C.MORRIS

p.3 NAME: Key

QIII

1/x rec

4pts 17. If $\tan\theta = \frac{4}{3}$ and $180° < \theta < 270°$, find $\sec\theta$

$\tan^2\theta + 1 = \sec^2\theta$ $\quad \frac{25}{9} = \sec^2\theta$ $\quad \sec\theta = \pm \frac{5}{3}$

$\frac{16}{9} + 1 = \sec^2\theta$ $\quad \boxed{\sec\theta = -\frac{5}{3}}$

5pts 18. Graph $y = 25\sin\left(\frac{1}{3}x\right)$ See graph paper

5pts 19. Graph $y = -8\cos(x - \pi/4)$ See graph paper

5pts 20. Find the exact value of cos75° using the addition formulas.

$\cos(45° + 30°)$ $\quad \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

$\cos 45° \cos 30° - \sin 45° \sin 30°$

$\frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2}$

5pts 21. Use addition formulas to verify $\cos 2\theta = 2\cos^2\theta - 1$

$\cos(\theta + \theta)$

$\cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) = \boxed{2\cos^2\theta - 1}$

4pts 22. Find all solutions on $[0, 2\pi]$ for $\cos^2\theta + 7\cos\theta + 6 = 0$

$(\cos\theta + 6)(\cos\theta + 1) = 0$

$\cancel{\cos\theta = -6}$

$\cos\theta = -1$ $\quad \boxed{\theta = \pi}$

4pts 23. Find all solutions on $[0°, 360°]$ for $\cos^3\theta + \cos^2\theta = 0$

$\cos^2\theta(\cos\theta + 1) = 0$

$\cos\theta = 0$ $\quad \boxed{90°, 270°, 180°}$

$\cos\theta = -1$

$x^2 + y^2 = 1$

$\pi = 3.14$ radians

$\pi = 180°$

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

If in the form $A\sin(Bx + C)$ or $A\cos(Bx + C)$

$|A|$ = amplitude period = $2\pi/B$

phase shift = $-C/B$