ABSTRACT: The ideal functional electrical stimulation (FES) system requires a mathematical model to provide feedforward control of the stimulation parameters such that they are optimal for different individuals across a range of physiological conditions, muscles, and tasks. Recently we tested and validated such a model using able-bodied subjects. The purpose of this study was to determine whether this model applied to persons with spinal cord injuries (SCI). To this end, the isometric force responses of the paralyzed quadriceps femoris muscles of 14 adolescents and young adults were tested. For each subject, the force responses to two six-pulse stimulation trains were used to identify the parameter values of the model and then the model was used to predict the force responses to three train patterns across a range of frequencies in both a nonfatigued and fatigued condition. The intraclass correlation coefficients (ICCs) between the experimental and predicted force–time integrals and peak forces were above 0.90 for 12 of the 13 stimulation trains tested in the nonfatigued condition and all 13 trains tested in the fatigued condition. The success of our model with SCI subjects leads us to believe that our model may be useful for designing optimal stimulation parameters for standing and ambulation in patients who use FES.
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MATHEMATICAL MODEL THAT PREDICTS ISOMETRIC MUSCLE FORCES FOR INDIVIDUALS WITH SPINAL CORD INJURIES

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Despite over 40 years of investigations and advances, clinical applications of functional electrical stimulation (FES) as a neuroprosthesis for persons with severe central nervous system dysfunction are not widespread. In part, this is due to limitations in control strategy that lead to imprecise control of muscle force responses and rapid fatigue.[31]The frequency and pattern of the pulses within each stimulation train affect the force response and muscle fatigue. If optimal stimulation strategies that use both feedforward and feedback control are to be developed for each individual across a range of muscles, tasks, and physiological conditions, then mathematical models that accurately predict the forces in response to stimulation trains of various frequencies and stimuli patterns will be required.

Most of the physiology-based muscle models in the literature can be classified as either Huxley-type or Hill-type models. Huxley-type models evolved from Huxley's early crossbridge models[24] and are based primarily on biophysical and biochemical mechanisms of muscle contraction at the molecular level.[36] However, this type of model is generally too complicated to be incorporated into FES control systems that require rapid identification of the parameter values and the optimization of the force output.[1,35] Hill-type models use a motor, damper,

Abbreviations: CFT, constant-frequency train; DFT, doublet-frequency train; FES, functional electrical stimulation; FTI, force–time integral; ICC, intraclass correlation coefficient; PF, peak force; SCI, spinal cord injury; VFT, variable-frequency train

Key words: functional electrical stimulation; human quadriceps femoris; muscle fatigue

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and spring to represent the contractile, viscous, and elastic elements of the muscle, respectively.[17,35] Previously, we developed a Hill-type force model that incorporated a function for muscle activation so that the model could use stimulation patterns as inputs.[12] Our force model is governed by two differential equations:

$$\frac{dC_N}{dt} = \frac{1}{\tau_c}\sum_{i=1}^{n} R_i \exp\left(-\frac{t-t_i}{\tau_c}\right) - \frac{C_N}{\tau_c} \qquad (1)$$

with $R_i = 1 + (R_0 - 1)\exp[-(t_i - t_{i-1})]/\tau_c$ and

$$\frac{dF}{dt} = A\frac{C_N}{K_m + C_N} - \frac{F}{\tau_1 + \tau_2 \dfrac{C_N}{K_m + C_N}} \qquad (2)$$

Equation 1 represents the activation of the muscle by modeling the dynamics of the rate-limiting step leading to the formation of the Ca^{2+}–troponin complex (C_N, unitless), in which R_i (unitless) accounts for the nonlinear summation of the Ca^{2+} transient in single muscle fibers when stimulated with two closely spaced pulses (i.e., a doublet).[16] Equation 2 represents the development of the mechanical force (F, Newton) and was derived from a linear spring, damper, and motor in series. This equation is driven by force-producing crossbridges and mediated by a Michaelis–Menten term, $C_N/(K_m + C_N)$. The force model is governed by six parameters: τ_c, R_0, A, K_m, τ_1, and τ_2. The definitions of symbols used in the above equations are detailed in Table 1. This force model has been rigorously tested with data collected from quadriceps femoris muscles of healthy, young adults under a variety of physiological conditions.[12] It successfully predicted the force responses to stimulation trains over a wide range of stimulation frequencies, pulse patterns, number of pulses, and durations when muscles were both nonfatigued and fatigued. However, we have not previously tested the model using data collected from patient populations.

Following spinal cord injury (SCI), paralyzed muscles may experience dramatic muscle atrophy[8] and muscle fiber-type transformation towards predominance of type II fibers.[2] Thus, the strength, contractile, and fatigue-resistance characteristics of the paralyzed muscles of persons with SCI differ from those of able-bodied individuals. Furthermore, the quadriceps muscles of persons with SCI display a higher interindividual variability in contractile characteristics and fatigue resistance than able-bodied persons.[22] Consequently, although our mathematical model was successful in able-bodied subjects, it needs to be tested for persons with SCI.

Table 1. Definition of symbols.

Symbol	Unit	Definition
C_N	—	Ca^{2+}–troponin complex
R_0	—	Mathematical term characterizing the magnitude of enhancement in C_N from successive stimuli
τ_c	ms	Time constant controlling the rise and decay of C_N
n	—	Total number of stimuli in the train before time t
t_i	ms	Time of the ith stimulation pulse
F	N	Instantaneous force
A	N/ms	Scaling factor for force and shortening velocity of the muscle
K_m	—	Sensitivity of strongly bound crossbridges to C_N
τ_1	ms	Time constant of force decline in the absence of strongly bound crossbridges
τ_2	ms	Time constant of force decline due to the extra friction between actin and myosin resulting from the presence of strongly bound crossbridges

We hypothesize that a more recent version of our isometric mathematical model,[12] which is capable of predicting responses not only to short trains but also to trains that contain up to 50 pulses for able-bodied persons, will accurately predict force responses to long stimulation trains for SCI patients. The purpose of this study was to test that hypothesis across a wide range of stimulation frequencies and to three patterns of stimulation in both nonfatigued and fatigued conditions using data collected from the paralyzed quadriceps femoris muscles of persons with SCI.

METHODS

Fourteen subjects (12 male, 2 female) with complete paralysis from spinal cord injuries (American Spinal Injury Association, classification A or B) and with a mean age of 17 years (range, 10–21 years) participated in this study. The mean interval after injury was 4.4 years (range, 0.8–16 years). Six subjects had implanted lower-extremity electrodes to the quadriceps femoris muscles, although only three currently used the electrodes to condition their muscles or for FES. One other subject used surface electrodes to condition his muscles. All except one subject had thoracic-level motor-complete spinal cord injuries. Before testing, each subject and their legal represen-

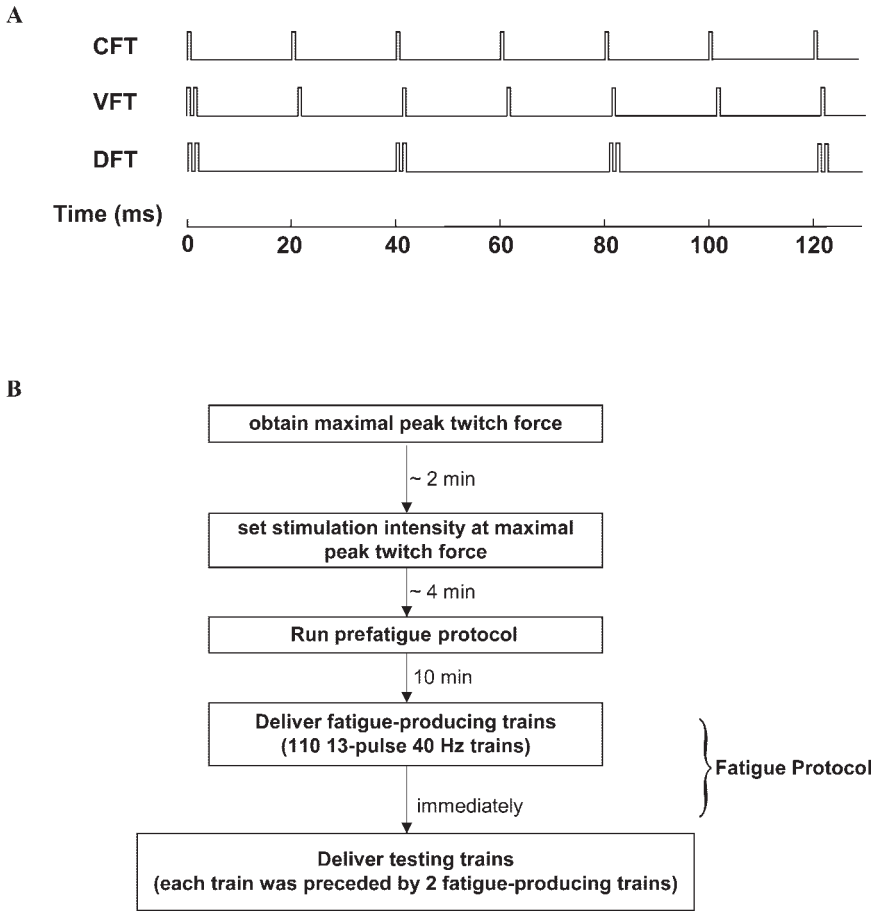

FIGURE 1. Schematic representations of the three pulse patterns **(A)** and experimental procedures **(B)** used in this study (see text for details). **(A)** Top line: a 50-Hz constant-frequency train (CFT) with all the pulses spaced equally within the train; middle line: a 50-Hz variable-frequency train (VFT) with an initial 5-ms interpulse interval (doublet) followed by equally spaced pulses; bottom line: a 50-Hz doublet-frequency train (DFT) with doublets throughout the train.

tative (if a minor) reviewed and signed a consent form that was approved by both the University of Delaware and Temple University Human Subject Review Boards.

Isometric force data were collected from quadriceps femoris muscles. The experimental protocol was similar to that described previously.[12] Briefly, subjects were seated on a force dynamometer (Kincom III 500-H; Chattecx, Chattanooga, TN) with their hips flexed to about 75° and their knees flexed to about 90°. Two 7.5 cm × 12.5 cm self-adhesive electrodes were used for transcutaneous electrical stimulation of the muscle. One electrode was placed distally over the muscle belly of the vastus medialis and the other was placed proximally over the rectus femoris muscle. The force transducer was positioned against the anterior aspect of the leg, proximal to the lateral malleolus. Isometric force data from the dynamometer's force transducer were digitized at a rate of 200 Hz and stored on a personal computer using customized software (Labview 5.1, National Instruments Corporation, Austin, TX).

Experimental Testing. The experimental procedures were identical for all 14 subjects (Fig. 1B). First, the peak twitch force of the subject's quadriceps muscle was recorded when stimulated with a series of single 600-μs pulses delivered at a rate of 1 every 10 s as the stimulator voltage was incrementally increased until the maximum of 150 volts (Grass Stimulator S88; Grass Instruments, West Warwick, RI). The peak twitch forces ranged from 31 to 165 N. Next, the stimulation intensity was adjusted using a 1-s, 20-Hz stimulation train to produce a force equal to the subject's maximum recorded peak twitch force. The intensity was kept unchanged throughout the remainder of the testing session. The work of Gerrits and colleagues[21] and our own unpublished observations indicate that the paralyzed quadriceps muscles of SCI subjects have elevated twitch re-

sponses compared to able-bodied subjects. The elevated twitch response of SCI subjects leads to a twitch-to-tetanus ratio for the quadriceps muscle of approximately 0.25, which corresponds to 0.13 in able-bodied subjects.[21] We believe, therefore, that the 1-s, 20-Hz train was producing approximately 25% of the force-generating ability of the muscle. Based on the force–frequency relationship of the paralyzed quadriceps muscles (unpublished observation), the 1-s, 20-Hz train produces approximately 65% of the force produced in response to a 100-Hz train. Thus, we estimate that in response to the higher frequencies tested, we were recruiting approximately 35% to 40% of the force-generating ability of the muscles.

Each subject received a prefatigue and a fatigue protocol. The muscles were not potentiated before the prefatigue protocol because pilot testing showed that activating the muscle to induce potentiation also caused fatigue for the SCI subjects. Each stimulation train was limited to either 1 s in duration or 50 pulses, whichever yielded the shorter duration of stimulation, to avoid fatigue within each stimulation train. In the prefatigue protocol, stimulation trains were delivered at a rate of one train every 20 s to avoid fatigue. Three different patterns of stimulation trains were tested at multiple frequencies. Traditionally, during FES, skeletal muscles have been activated with constant-frequency trains (CFTs), in which the pulses within the train are equally spaced (Fig. 1). Recent studies in our laboratory showed that stimulation trains that contained closely spaced pulses (doublets) at the beginning of a train [i.e., variable-frequency trains (VFTs), Fig. 1] or throughout the train [i.e., doublet-frequency trains (DFTs)] produced greater isometric forces from fatigued human skeletal muscles and greater power during dynamic contractions than CFTs[4,5,11,25] and consequently may be more suited to FES applications than CFTs. Although we anticipated that there would be little difference in the force responses to the three train types at the higher frequencies tested, at the lower frequencies the force responses to the three train types can be expected to have very different shapes, and therefore provide a good test of the model's ability to predict the shape of the force response. The testing trains were CFTs at 12.5, 33, 50, 80, and 100 Hz, and VFTs and DFTs at 12.5, 33, 50, and 80 Hz (13 total testing trains). Additionally, a twitch was delivered. The doublet pulses in the VFTs and DFTs were separated by 5 ms. The frequencies given for the VFTs and DFTs are the mean frequencies of the entire stimulation train. The 13 testing trains and the twitch were delivered in a randomized order and then repeated in the reverse order. The force responses to the two occurrences of each train were averaged for data analysis. The same randomized order was used for all subjects.

Following a 10-min rest, the fatigue protocol started with 110, 13-pulse, 40-Hz trains to produce fatigue (fatigue-producing trains) followed by the 13 testing trains and the twitch. The 13 testing trains and twitch were then repeated in reverse order (Fig. 1). As in the nonfatigued condition, the force responses to the two occurrences of each train were averaged for data analysis. Each testing train was preceded by two 13-pulse, 40-Hz trains to maintain a steady state of fatigue. Stimulation trains were delivered with a 700-ms rest time between successive trains.

Simulation. The forces produced by the 50-Hz CFT and 12.5-Hz VFT stimulation trains were used to identify the model parameter values for able-bodied subjects in our previous work.[13] In this study we also used the force responses to these two trains to identify the model parameter values. The force model has six parameters, τ_c, R_0, A, K_m, τ_1, and τ_2, that need to be identified for each subject. A pilot study using the same procedure as that used to find the parameter values for able-bodied subjects resulted in poor performance of the model for SCI subjects. We also found, however, that only modifying the procedures for identifying the values for parameters τ_c and R_0 improved greatly the performance of the model. This was probably because of the faster contractile speeds of paralyzed muscle than nonparalyzed muscle.[21] For able-bodied subjects parameter τ_c, the time constant controlling the rate of rise and decay of C_N was fixed at 20 ms,[13] whereas for SCI subjects τ_c was calculated by multiplying the subject's half-relaxation time by 0.22. The factor of 0.22 was derived from data of able-bodied subjects by dividing the τ_c value (20 ms) by the subjects' average half-relaxation times (~90 ms). We found through several experiments that scaling τ_c to each SCI subject's half-relaxation time greatly improved the fit of the model to the experimental data. Each subject's half-relaxation time was defined as the average of the time period during which force declined 50% from its maximum in response to the 50-Hz CFT and the 12.5-Hz VFT. In addition, the relationship $R_0 = K_m + 1.04$ used for able-bodied subjects[13] failed for SCI subjects. In this study R_0, a mathematical term characterizing the magnitude of enhancement in C_N from successive stimuli, was a free variable.

Parameter τ_1 is the time constant characterizing the relaxation of the muscle due to the absence of

strongly bound crossbridges ($C_N/(K_m + C_N)$). Similar to our previous studies,[11,12] τ_1 was calculated using the force decay (between 50% and 25% of the peak forces) at the end of the forces produced by the 50-Hz CFT and 12.5-Hz VFT. By setting the term $C_N/(K_m + C_N) = 0$, eq. (2) is reduced to $dF/dt = -F/\tau_1$, and values of τ_1 were calculated separately for the 50-Hz CFT and the 12.5-Hz VFT forces by performing a least-square fit of $\ln F$ versus time t. The average was taken as the τ_1 value for each subject.

When muscles were nonfatigued, the remaining four parameters were identified using an objective function (G_{nf}) as in

$$G_{nf}(A,R_0,K_m,\tau_2) = \sum_p (F_{pred}(t_p;A,R_0,K_m,\tau_2) - F_{\exp}(t_p))^2 \quad (3)$$

F_{pred}, a function of parameters A, R_0, K_m, and τ_2, represents the force predicted by eqs. (1) and (2) at time t_p; $F_{\exp}$ represents the experimental force at time t_p; p is the number of force data points. The objective function G_{nf} was minimized using feasible sequential quadratic programming in C (CFSQP), which identifies the optimum values for the free variables numerically.[26] Then, the model was used to predict the forces to the stimulation trains tested in this study (see Fig. 2 for examples).

For fatigued muscles of able-bodied subjects, parameters τ_c and τ_2 could be fixed at the prefatigued values.[11,12] Pilot work with SCI subjects showed that, in addition to τ_c and τ_2, parameter R_0 could also be fixed at the prefatigued value. Thus, there were only three free parameters that needed to be identified when muscles were fatigued. Parameter τ_1 was identified as described above using the force response of the fatigued muscle. Parameters A and K_m were identified using

$$G_{\text{fat}}(A,K_m) = \sum_p (F_{pred}(t_p;A,K_m) - F_{\exp}(t_p))^2 \quad (4)$$

The objective function G_{fat} was also minimized using CFSQP.[26] Then the parameterized model was used to predict the force responses to the stimulation trains tested in this study.

Data Management and Analysis. ***Fatigue Index.*** To evaluate the degree of fatigue, a fatigue index was calculated as

$$\text{Fatigue Index} = \left(1 - \frac{PF_{110}}{PF_1}\right) \times 100\% \quad (5)$$

PF_{110} and PF_1 represent the peak forces of the last (i.e., 110th) and the first fatigue-producing trains, respectively. Larger values of the fatigue index indicate that the muscle was more fatigued.

Screening Data. Experimental force responses were averaged over the two occurrences for each testing train to minimize the influence of fatigue and potentiation. Fatigued data from 2 of the 14 subjects were too noisy, due to low forces, to be analyzed and therefore were excluded. Thus, there were 14 nonfatigued and 12 fatigued muscles analyzed. In addition, because the subjects' muscles were weak due to atrophy and the fatigue protocol caused more than a 50% decline in force for most subjects, the force responses to some testing trains, especially low-frequency trains, became too noisy and unreliable. One of the authors (WBS) screened the experimental data and data were excluded if the peak force was lower than 10 N, if there was a baseline shift, or if a reflex or spasm occurred during activation. The number of subjects, therefore, was fewer for some testing trains.

Data Analysis. We noted that there were no significant differences in either the fatigue indices or contraction speeds (τ_c) between subjects with implanted electrodes and those without implanted electrodes (mean fatigue indices: implanted = 54.8%, nonimplanted = 71.6%; τ_c: implanted = 12.7 ms, nonimplanted = 12.7 ms) or between those subjects that used electrical stimulation to condition their paralyzed muscles and those who did not use electrical stimulation (mean fatigue indices: conditioned = 48.8%, nonconditioned = 70.7%; τ_c: conditioned = 12.2 ms, nonconditioned = 12.9 ms). Thus, all the subjects' data were grouped together for analyses. The accuracy of the model's predictions was evaluated in three ways. The force profile as a function of time for each testing train was compared between the experimental and predicted forces using a Pearson coefficient of determination (r^2) to compare the experimental and predicted forces recorded at each time point sampled (i.e., every 5 ms). The coefficient of determination estimates the variance in the experimental data that can be accounted for by the predicted forces of the model.[29] A perfect match between the experimental and predicted force profiles would yield an r^2 of 1. However, the Pearson coefficient of determination is not sensitive to vertical offset (i.e., differences in force magnitude) between the experimental and predicted data. Intraclass correlation coefficients (ICCs), which are sensitive to any offset,[29] were therefore calculated to measure the agreement between the experimental and predicted force–time integrals (area under the

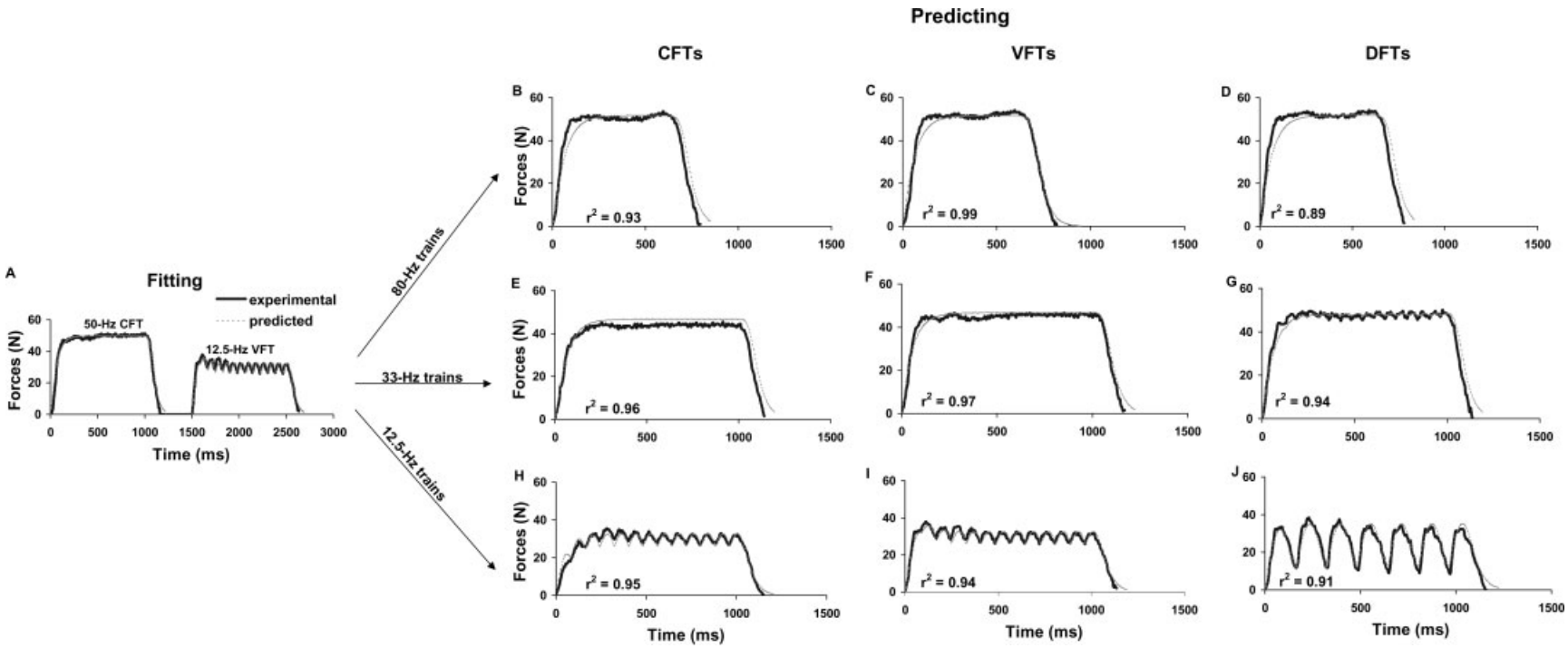

FIGURE 2. Example of predicted and experimental force responses to a wide range of stimulation frequencies from a typical subject when the muscle was not fatigued.

force–response curve; FTI) and peak forces (PF) for each testing train. High r^2 values but corresponding poor ICCs for the predicted and experimental peak forces and force–time integrals would be indicative of the model correctly predicting the shape of the experimental force response, despite a significant offset in the force profiles. Percentage differences were also calculated between the experimental versus predicted FTIs and PFs in response to each testing train by subtracting 1 from the ratio of the predicted to the experimental values and multiplying the difference by 100. A positive percentage difference indicates an overestimation of the model and a negative one indicates an underestimation. Histograms were constructed to evaluate the distribution of the percentage differences.

Parameter Values. Because our model is physiologically based, the parameters are associated with physiological elements of force generation. By comparing the parameter values between SCI and able-bodied subjects, changes in the paralyzed muscle after injury could be explored. Thus, independent *t*-tests with unequal variances were used to compare each of the parameter values between SCI and able-bodied subjects for the nonfatigued condition. Such analysis was not conducted for fatigued muscles because the level of fatigue was not comparable between these two groups of subjects. The nonfatigued data for able-bodied subjects were taken from our previous study and remodeled using the R_0–Km relationship.[13] Comparisons were considered significant if P values were < 0.05.

RESULTS

The averaged r^2 values between the experimental and predicted force profiles were above 0.90 for the nonfatigued testing trains except at 12.5 Hz (r^2 = 0.74–0.79) and above 0.80 for the fatigued testing trains except the 12.5-Hz CFT (r^2 = 0.76), suggesting that the model accurately predicted the shape of the force profile for a wide range of stimulation frequencies regardless of train type (Figs. 2 and 3). The ICC values between the experimental and predicted force–time integrals and peak forces were above 0.90 for 12 of the 13 trains tested in the nonfatigued condition and all 13 trains tested when the muscles were fatigued, suggesting a strong agreement between the experimental and predicted FTIs and PFs in both nonfatigued and fatigued conditions. The accuracy of the model's predictions permitted the construction of force–frequency relationships for both the FTIs and PFs for the paralyzed nonfatigued and fatigued muscles that were largely accurate for the three train types (Fig. 4). Larger standard errors were seen with fatigued muscles, which suggested greater variability in force across subjects in fatigued muscles. This increased variance in fatigued muscles may be due to a substantial difference in the amount of fatigue produced by our fatigue-producing trains between subjects. By the end of the fatigue-producing trains, subjects' force declined by approximately 68% (median), ranging from 17% to 80%.

It was possible to make 163 and 127 individual comparisons between the experimental and predicted forces for the nonfatigued and fatigued muscles, respectively. When muscles were not fatigued, the model's predicted FTIs and PFs were within 10% of the experimental forces for 89% and 81% of the comparisons, respectively (Fig. 5). The averaged nonfatigued peak force produced by the stimulation trains was 93 N. Thus, 10% error is equivalent to only 9 N. When muscles were fatigued, for 79% and 74% of comparisons, the model's predictions were within 15% of the experimental data for the FTIs and PFs, respectively. The averaged peak force of fatigued muscles was 33 N; 15% error is therefore about 5 N.

Parameter A was significantly lower ($P < 0.001$) for the SCI (mean = 1.55) than for the able-bodied subjects (mean = 3.86). SCI subjects had larger R_0 (mean = 9.9) than able-bodied subjects (mean = 1.2; $P < 0.001$). SCI subjects also had smaller τ_c (mean = 12.7) than able-bodied subjects (all = 20; $P < 0.001$). Interestingly, no significant difference was detected for parameters K_m, τ_1, and τ_2 between the SCI (means = 0.17, 39, 41, respectively) and able-bodied subjects (means = 0.19, 50, 45, respectively).

DISCUSSION

Our mathematical model successfully predicted the force responses to three different stimulation patterns over a wide range of frequencies for patients with SCI. The model's predictions accounted for more than 90% and 80% of the variance of the experimental force profiles for the majority of the testing trains when the muscles were not fatigued and fatigued, respectively. The predicted force–time integrals and peak forces correlated well with the experimental FTIs and PFs with ICCs above 0.95 for most of the testing trains. Consequently, the model successfully predicted the force–frequency relationships for both nonfatigued and fatigued muscles to the three different train types (Fig. 4).

The quadriceps femoris muscle of able-bodied adults contains approximately 40% type I fibers and

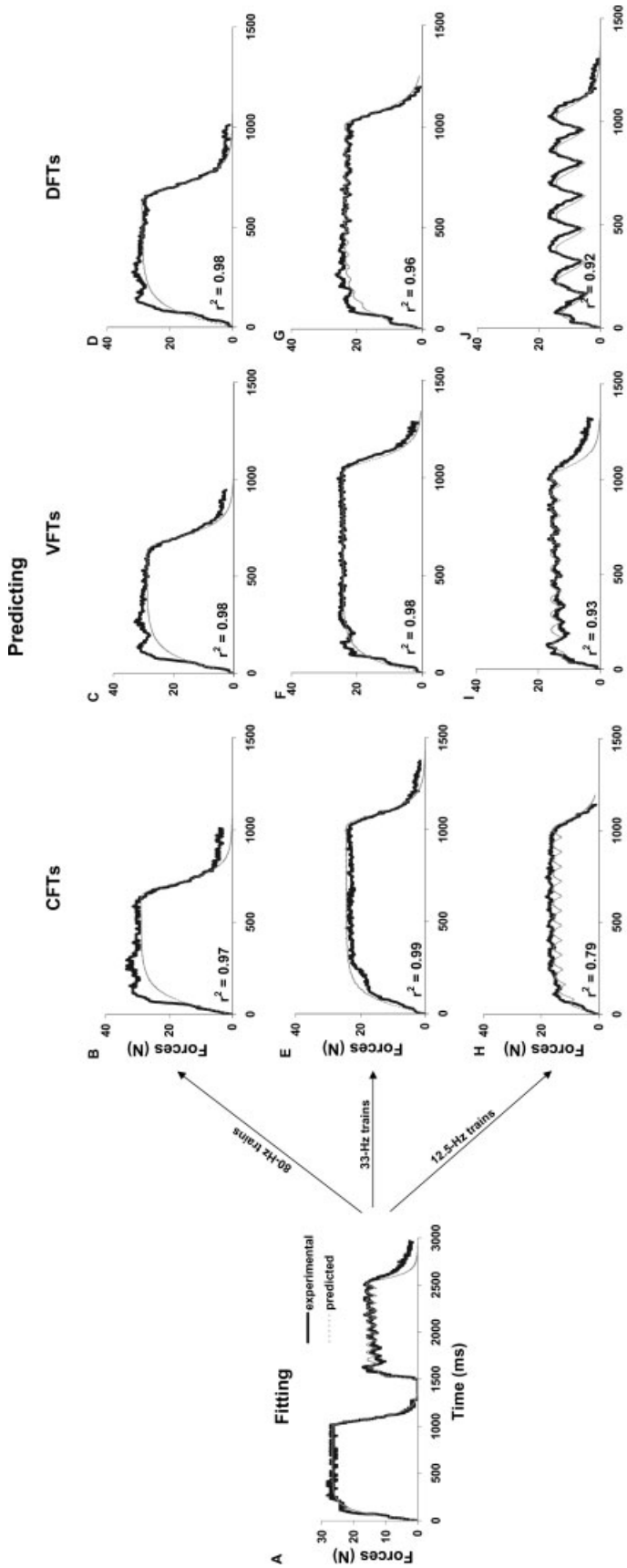

FIGURE 3. Example of predicted and experimental force responses to a wide range of stimulation frequencies from the same subject as in Figure 2 when the muscle was fatigued.

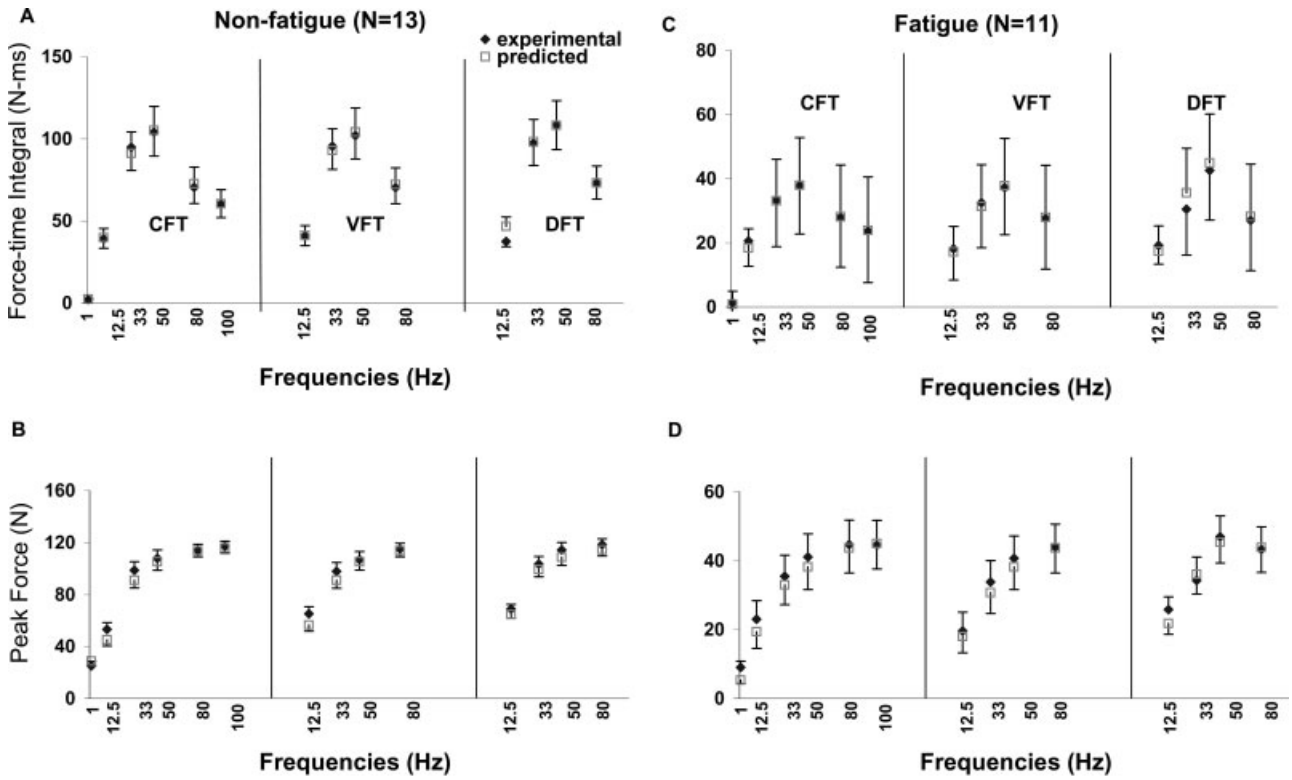

FIGURE 4. Comparison between experimental and predicted force–time integrals **(A, C)** and peak forces **(B, D)** at each frequency of CFTs, VFTs, and DFTs for nonfatigued muscles **(A, B)** and fatigued muscles **(C, D)**. Bars represent standard errors.

60% type II fibers.[33] After injury, paralyzed muscles typically gradually transform from type I to type II fibers, resulting in a predominance of fast fatigable muscle fibers.[2,32] Fast, type II fibers have faster calcium dynamics, resulting from both faster calcium release[18] and uptake,[19] which is consistent with the much smaller τ_c values for SCI than for able-bodied subjects. Parameter A, the scaling factor, is also significantly smaller for SCI subjects, which is consistent with lower forces due to muscle atrophy after SCI injury.

Parameter R_0 represents the nonlinear summation of the Ca^{2+} transient in single muscle fibers when stimulated with a doublet,[16] which is thought to account for the augmentation in force produced by stimulation trains that contain doublets (i.e., VFTs and DFTs).[16] Compared to data of able-bodied subjects,[13] our model showed a larger value of R_0 for SCI subjects. This is contradictory to the experimental observation of a greater augmentation by doublet force in type S units than in potentiated, type F units of the cat.[7] However, Griffin and colleagues[23] have shown that the paralyzed human thenar muscles of subjects with SCI show a greater augmentation of the force response to doublets than able-bodied subjects, suggesting that persons with SCI have a larger R_0 value. Additionally, the able-bodied subjects' data used for comparison were highly potentiated,[13] whereas we did not potentiate the SCI subjects' muscle before testing in this study because pilot work showed that stimulating the muscle to produce potentiation also produced fatigue for some subjects. Potentiation has also been shown to attenuate the augmentation in doublet force over single twitches.[10]

We anticipated smaller values for parameters K_m (greater Ca^{2+} sensitivity),[6] τ_1, and τ_2 (faster relaxation speed)[21] for SCI subjects, but our results showed no difference in these parameter values between the two subject populations. The lack of differences could be a result of differences in experimental procedures, mainly potentiation before testing for the able-bodied but not for the SCI subjects, or a compensation effect among parameters, or both. Potentiation has been shown to increase the Ca^{2+} sensitivity of the muscle.[34] Thus, the highly potentiated muscles of able-bodied subjects may have similar K_m values as the SCI subjects because the potentiation of the able-bodied subjects' muscles has a similar effect on K_m as the greater percentage of type II fibers does in the latter. Because potentiation does not affect the contractile speed of the muscle, including the relaxation times (τ_1 and τ_2),[3]

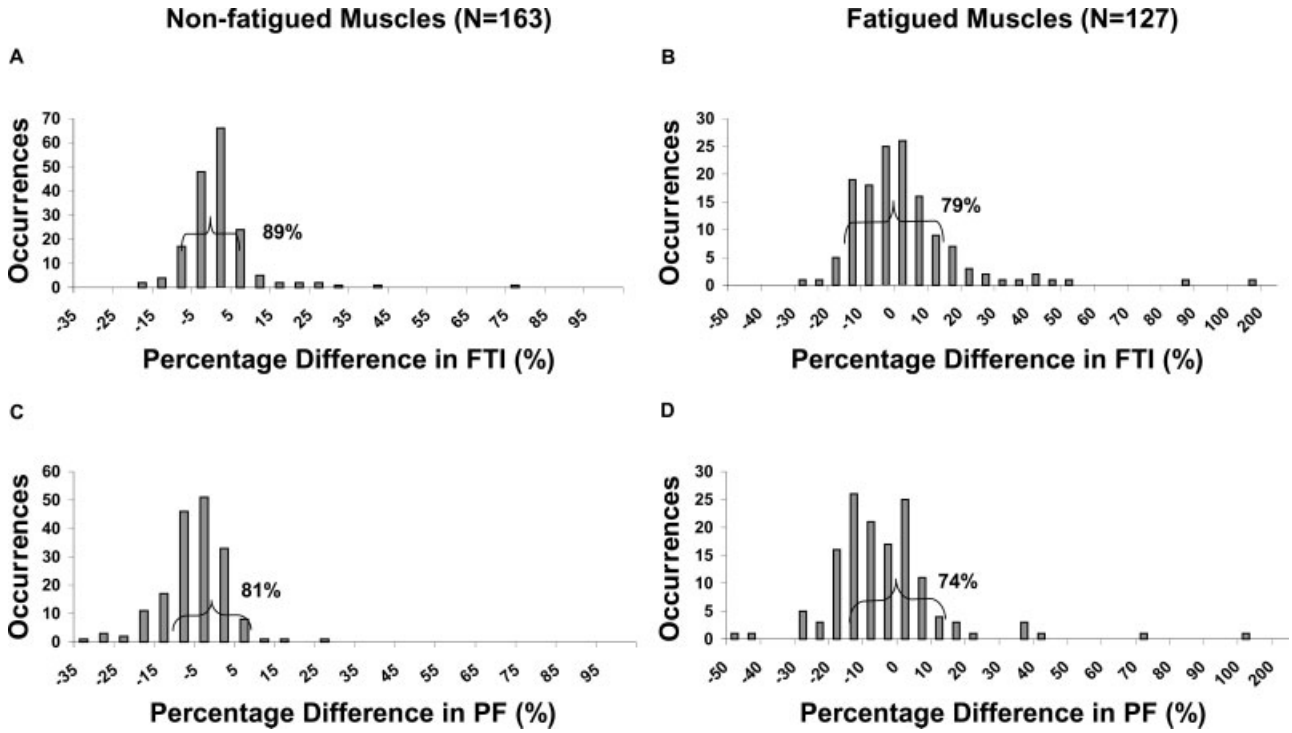

FIGURE 5. Histograms to compare the percentage differences between the predicted and experimental force–time integrals (FTI; **A, B**) and peak forces (PF; **C, D**) for nonfatigued muscles **(A, C)** and fatigued muscles **(B, D)**. For nonfatigued muscles 89% and 81% of the comparisons were within ± 10% for the FTI and PF, respectively. For the fatigued muscles 79% and 74% of the comparisons were within ± 15% for the FTI and PF, respectively.

the primary explanation for similar values for τ_1 and τ_2 between our SCI and able-bodied data is the compensation effect between τ_c, τ_1, and τ_2.

Despite the variance in the subjects' characteristics, such as time postinjury, age, and amount of fatigue produced by the fatiguing protocol, the model accurately predicted the force responses to stimulation trains with frequencies ranging from 1 to 100 Hz and CFT, VFT, and DFT train types. A few mathematical models have been developed and tested for SCI subjects to study muscle force[9] and fatigue.[20, 30] However, none showed the predictive ability of the model when the activation pattern was changed. The mathematical model presented in this article was originally developed for able-bodied subjects.[12] Despite the significant differences in quadriceps femoris muscles between the SCI and able-bodied subjects regarding muscle strength and fiber type composition, the model with minor modifications of the procedure used to identify parameter values, performed equally well for SCI subjects, suggesting that the model is robust. Also, we have recently developed a mathematical model that predicts the amount of fatigue produced in the nonparalyzed quadriceps muscles of able-bodied subjects when activated isometrically by stimulation protocols with different frequencies of stimulation or rest times between contractions.[14,15]

For either the nonfatigued or fatigued condition, the model only needed force responses to two brief stimulation trains to identify the parameter values and then to predict force responses to stimulation trains with a wide range of frequencies and patterns. The predictive ability and structural simplicity make our model favorable for FES applications by allowing rapid parameter value identification for each patient, fast optimization analysis of the system to identify the optimal stimulation schemes, and accurate predictions for feedforward control. We do, however, recognize that the present model only successfully predicted the force responses for quadriceps femoris muscle at one length under isometric conditions at one knee joint angle. Recent studies in our laboratory have shown that our model can predict isometric force responses at different knee joint angles[27] and the force responses throughout the range of motion during isovelocity contractions[28] when able-bodied subjects' quadriceps muscles were stimulated with frequencies and patterns similar to those presently studied. Thus, we envision that future versions of our model will be incorporated into a FES real-time control system that will improve the perfor-

mance of FES for standing and ambulation in individuals with SCI.

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