

An Unprudential Method

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Holding accurate beliefs about the past, present and future state of the world around us is of central importance to any rational human. Beliefs, however, are often vague and contradictory; rationality demands a systematic method for distinguishing and clarifying the nature of beliefs. This demand is made not because rationality is concerned with awareness of the truth as an end in itself; rather, the rational individual must employ such a system because it is to his benefit to be aware of the truth. Such systems of rationality are elusive, however, and often difficult to articulate. A philosophically popular response to this problem is known as a "subjective probability calculus." Loosely construed, a subjective probability calculus takes the degree to which people are confident of various propositions and translates those into probabilities. There are two primary areas of investigation possible here, one being whether humans do think about propositions in terms of a subjective probability calculus and the other being whether humans should.¹ This essay will investigate the latter issue (what I will call "normative subjective probability")—whether self-evaluation in terms of one's probabilistic degrees of belief is a good strategy when dealing with the real world. This does not require any claims about people's actual psychological organization.

This essay will, moreover, argue that closer attention to the principles of subjective probability does not necessarily make a person more rational, and hence may not have a place in our everyday lives. Such can very effectively make a person more coherent, but "fixing" one's personal beliefs in such a way as to be coherent might not be desirable. Additionally, it is unclear how useful probabilistic evaluation is except in rather particular or unrealistic circumstances. This essay will address an apparent solution to at least some these issues, in the form of Helen Beebe and David Papineau's relative probability theory, where the applications of such "fixes" are both practical and beneficial. Beebe and Papineau will be shown to have problems, however, in their insistence on how usefully a subjective probability calculus can be employed outside theoretical scenarios.

Before delving into these issues, more needs to be said about what the theory of subjective probability is committed to: analyzing degrees of belief in terms of betting behavior. For any given proposition a person might hold, that certainty of those beliefs is ascertained by finding out the odds he would be willing to place in favor of that belief. His degree of belief is the least favorable odds he would offer in a bet about the belief. So, if a person says he is willing to pay at most 60 cents in order to be paid a dollar if it rains tomorrow (and nothing if it doesn't rain), his confidence in the proposition that it will rain tomorrow is said to be 0.6. At the same time, it is implied the rational person with a confidence of 0.6 would also be willing to buy bets at any rate lower than 0.6 (since your margin of gain goes up).² The idea is that a rational person will want to organize his beliefs such that he can on average come out on top—or at least even—when faced with a bet. Of course, a person might not value money too much (perhaps he is a Franciscan who took an oath of poverty), or the bet might be the kind of thing that cannot be paid even if won (say, a wager that the world will end tomorrow), and so

the system of belief-determination has to be reframed in terms of hypothetical expected utility.³ Hypothetical expected utility works under the assumption that *something* valuable and divisible to the bettor will be spent and paid given a theoretical bet.

The move made in normative subjective probability is to claim that coherence must be maintained in a rational being via employing a subjective probability calculus. The normative force of this is that, if one's probability calculus is not properly set up, then one is susceptible to abuse by what is known as the "Dutch Bookie," which is an objectively bad thing. The upshot of having incoherent beliefs and making a set of bets with the Dutch Bookie is that you will always (not just on average) lose utility. Knowingly losing utility, indeed, seems irrational. An example of an incoherent belief set that will make a person susceptible is when two mutually exclusive beliefs whose probabilities sum to greater than 1 (that is, $p(A) + p(B) > 1$, given that A and B are mutually exclusive). The Dutch Bookie, who is himself always coherent, will take both bets, and will always gain utility. For example, suppose my confidence that George will be in Wal-Mart tomorrow at noon is 0.6, and my confidence that George will be in Safeway tomorrow at noon is also 0.6. The Dutch Bookie then buys both bets—in each bet, the Bookie gets, say, \$0.60 from me if he wins and I get \$0.40 from him if I win. Since George can only be in one place at one time, the Bookie will always earn at least \$0.20, since I can only possibly win on 1 of the 2 bets and the Bookie's bet is more valuable than mine by \$0.20.⁴

Yet arguably, the mere possibility of losing utility to the Dutch Bookie does not mean that holding incoherent beliefs like the above is irrational. That's because a person could simply refuse to make two opposing bets at the same time. The losses to Dutch Bookies only occur when the *whole book* of bets is purchased. A stipulation must then be added to the argument for the rationality of subjective probability—that a person who holds a set of beliefs must be willing to take all the bets concerning those beliefs.⁵

This recommendation which normative subjective probability seems to impose on the supposedly rational man is a strange one, indeed. The underlying assumption is that rationality must be used to defend individuals against trickery. This seems intuitive enough; after all, since everyone wants to maximize his utility, and it would be irrational for a person to act in such a way that he could be fooled. But the caveat, "Never make books," would serve this purpose as well. The constraints that the rule, "You should take all bets concerning your beliefs," moreover, put on a person's beliefs (that a person must be willing to take all bets concerning them) seem too strong to be used practically. Certainly, the rule forces a person to rearrange their beliefs in a protective manner. But what is the justification of imposing that constraint in the first place? It appears that normative subjective probability is a method to preserve the conclusion that maintaining a coherent calculus is rational.⁶ Yet this seems to be a circular argument. The rule, "You should take all bets concerning your beliefs" is justified because it is necessary to prove probabilistic coherence is rational; the notion that probabilistic coherence is rational is in turn justified by the requirement that people take all bets.

A way out of this criticism would be to fall back on a descriptive claim—people are, actually, always willing to take multiple bets on beliefs concerning related propositions. If that is the case, the above constraint would be reasonable.

However, this seems to be a very strong psychological claim—too strong to be ever proven by evidence (even if people have *always* been willing to buy books of bets, they still might reasonably refuse in the future). Moreover, there is a great deal of evidence that people are fallible when it comes to maintaining beliefs (that's the reason proponents of normative subjective probability want to recommend coherence, after all), to such an extent that it seems quite plausible a person steadfastly believe a coin has a 60 percent chance of landing heads, a 60 percent chance of landing tails, and still refuse to even hypothetically take bets on both at the same time.⁷

Rational beings also end up, by the general conception in normative subjective probability, as quite unlikable people—Dutch Bookies themselves! An underlying notion of the Dutch Bookie is that he is, in fact, quite a rational person; he seeks out the best way to derive utility out of people via tricky betting. He is the completely coherent one taking advantage of the poor irrational saps out there; the Dutch Bookie does all the probabilistic calculations right—and benefits with large rewards. Yet he's a very nasty fellow, we surmise, one who does not care at all about the utility of others and has no dignity of his own. Normative subjective probability has no guard against this unfortunate consequence of its recommendations; instead, it seems the best way to beat the Dutch Bookies is to become a Dutch Bookie. The widely accepted conception of the rational being as utility-seeking combined with the inherently combative nature of betting (someone always loses) leads to this unfortunate conclusion. The rule "don't take advantage of irrational people" would serve to prevent this transformation, but problems still emerge whether such a rule is rational (in which case reason contradicts the recommendations of normative subjective probability) or irrational (in which case it is a questionable proposition to begin with).

Another issue is that problems of incoherence only actually emerge when a person attempts to organize his beliefs in terms of probabilities in the first place.⁸ Suppose a person believes Alex is smarter than Joan, and Joan is smarter than Beatrice—but also that Beatrice is smarter than Alex. These are obviously incoherent positions taken together, but it might not bear on a person's life at all until they are actually challenged to clarify their positions. In other words, asking the question (perhaps of oneself), "is Alex smarter than Joan?" creates the problem of incoherence; previously, the beliefs held were, perhaps, vague and the person was unlikely to make any serious commitment to them. Normative subjective probability creates its own problem, then, and is its own solution, making the theory practically useless.

A related problem is normative subjective probability's inability to rationally recommend how to resolve such conflicts. It is equally rational in the above position to revise one's opinions and say "Beatrice is actually dumber than Alex" as it is to say, "Joan is actually dumber than Beatrice." The "rational" solution seems almost arbitrary.⁹ In fact, one could actually come up with a sort of rotational epistemology; whenever the question is asked, "how do you rank preferences?" a person could respond with a different "fix"—the first time Joan is dumber than Beatrice, the second time Beatrice is dumber than Alex, etc. It could be argued that recalibrating your beliefs properly is the job of other rational capacities, and is not the domain of subjective probability; this seems right, but it also calls into question how useful subjective epistemology is in evaluating the rationality of beliefs. We

can rely on our truth-evaluative capacities to determine that, say, the proposition "Beatrice is smarter than Alex" was wrong; in fact, we might as well just rely on such capacities in all cases of self-evaluation as they will be necessarily employed anyway when normative subjective probability finds incoherence.

Combining such truth-evaluative capacities with subjective probabilities is, essentially, the project of Beebe and Papineau in their 1997 paper "Probability as a Guide to Life."¹⁰ Beebe and Papineau, who assume from the beginning that people's beliefs do, indeed, conform to a probability calculus,¹¹ argue that the prudential correctness of the beliefs of a person who doesn't have access to objective probabilities is still constrained by such probabilities.¹² Prudential correctness is importantly different from the notion "rational" espoused by other proponents of subjective probability (since rational beliefs might be blatantly false), but Beebe and Papineau nevertheless offer an argument for a more rational form of subjective probability, because a rational being will naturally seek to maximize his own utility.

Beebe and Papineau postulate that it is prudentially rational to follow the Relative Principle, which is defined as follows: "The correct subjective probabilities for an agent to attach to outcomes ... are the probabilities of the [outcomes] relative to the agent's knowledge of the set-up."¹³ In other words, how good a given subjective probability is depends crucially on the knowledge the individual possesses. The example given, and repeated often by Beebe and Papineau, is that of a coin toss. Alma tosses a coin onto a table, then covers it with her hand. It can be said, on the one hand, the coin is determinately either heads or tails (such is referred to as the single-case probability; the coin has either a 1 or a 0 probability of being heads, and the same for tails); but the relative probability of the coin being heads given Alma's *knowledge*—what she has seen and is aware of—is 0.5.¹⁴ This differs substantially from the above conceptions of subjective probability because it imposes additional, somewhat stringent requirements of rationality on an individual. Not only must rational beliefs be coherent, but they must also conform to the individual's knowledge. The relevant knowledge in the above proposition is that the coin is fair—given a large number of trials, the coin will end up as heads or tails an equal amount of the time. A denial of this proposition—such as a claim, say, there is a .9 probability of heads and .1 probability of tails—is imprudent (and hence irrational, as you'll lose money in bets) because coins really do tend to be fair.

The idea is that subjective probabilities can be good guides for decision-making. Beebe and Papineau are looking for ways to ensure maximal expected utility; this can only be gained, on their view, when a person conforms to the best rules under the epistemological conditions they find themselves in. A further point of contention—that we should pay attention to relative probabilities rather than single-case ones—is the actual focus of Beebe and Papineau's paper, but their significant point here is the insistence that applying relative, knowledge-state dependent conditions on a person's probability calculations is rational and prudent.

A problem emerges with Beebe and Papineau's theory when analogies to the above hand-covering coin example are brought to light. Take, for instance, the famous philosophical problem of other minds. Another person's mind, one can argue, is completely inaccessible to us—we can have no direct observation of a person's thoughts and feelings. We can never actually verify that another person

actually has a mind like our own. Though one might say there is in fact direct evidence in favor of other minds, suppose Robert believes such evidence is unobtainable. His friend Patricia, then, is like Alma's covered coin—though it is actually determined whether or not she has a mind, Robert believes he can never verify this. So, how should he proceed? Given Robert has no evidence either way, Beebe and Papineau seem to recommend he straddle the fence, in a way—treat her with a 0.5 probability that she actually has a mind like his own. Yet this doesn't seem like a very prudent way to act. In fact, it seems much more prudent to simply deny that probability theory is necessary and assign a value of 1 to the proposition that Patricia has a mind.

Beebe and Papineau argue around such examples in a similar fashion. Their position is that when there is no probabilistic law that can govern a person's decision-making skill (that is, no analogy in the case of other minds to the knowledge that coins tend to be fair), the Relative Principle cannot be applied. That might have been sufficient to explain the problem, but their claim is actually quite bold: in such cases, "there is no prudentially correct degree of belief."¹⁵ Yet assigning values of either 1 or 0—rather than taking a middle, incoherent position—seems to be a much more desirable outcome.

There seem, in any case, to be prudential demands threatening Beebe and Papineau's theory. These demands are related to the pitfalls of normative subjective probabilism: betting systems don't adequately argue for a person's actions in many practical situations.

An interesting example illustrating this point can be found in a recent *New Yorker* article by Elizabeth Kolbert.¹⁶ The article describes an intriguing incident involving a particle accelerator at Brookhaven. In a *Scientific American* letter to the editor, a physicist—while dismissing a reader's concern that a black hole might form—raised the possibility that the accelerator might generate "strangelets," matter normally found in neutron stars but which could, when formed, convert all nearby matter to more strangelets, and soon the world as we know it would vanish. The probability of such an event was very, very small, the physicist insisted in the letter, but the mere possibility was enough to lead to a *London Times* article with the headline, "Big Bang Machine Could Destroy Earth." Now, physicists working on such projects are instructed, in relation to the accelerators' "world-destroying potential, 'not to say the probability is very small but to say the probability is zero.'"¹⁷

The upshot of this example is that prudential outcomes and adherence to even relative probabilities given one's knowledge could be mutually exclusive. Hence, acting on accurate probabilities might not be an advisable activity. This is not only the case for external communications and exchanges (though betting scenarios fall under that category) but also internal probability-deliberations; having the peace-of-mind that there is a probability of 0 that the world will end because of particle accelerators may, in fact, be better than having a probability of .00000000000001. Admittedly, this does not directly attack Beebe and Papineau's argument about how in a strict sense (that is, disregarding external effects of probability calculations) adherence to relative probabilities is a good idea. However, the essence of their argument is that prudent beliefs match relative probabilities, and this notion is still vulnerable to the problems of normative subjective

probability in general—that good beliefs don't have to match subjective probabilities.

There are, then, methods of belief-evaluation not accounted for by subjective probability, which could actually bear more strongly in how we should arrange our beliefs. Coherence and relative probabilities may be very handy indeed in guiding a person through betting scenarios, but lose their usefulness when we incorporate the wide variety of beliefs people hold. It is also unclear if incoherence or inability (or unwillingness) to apply relative probabilities necessarily damages a belief under general circumstances. Those only become dire errors when philosophers imagine people in unrealistically strict and isolated scenarios; but the real world is far too complex for the theories to adequately advise people about how to organize their beliefs. □

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¹ Henry Kyburg, "Subjective Probability: Criticisms, Reflections, and Problems," *Journal of Philosophical Logic* 7 (1978): 157-180, p. 158.

² *Ibid.*, p. 159.

³ Alan Hajek, "Interpretations of Probability," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta (fall 2007). This entry can be found online at <http://plato.stanford.edu/archives/fall2007/entries/probability-interpret>.

⁴ *Ibid.*

⁵ Kyburg, *op. cit.*, p. 161.

⁶ *Ibid.*, pp. 161-2.

⁷ Hajek, *op. cit.*; see also Kyburg, *op. cit.*, pp. 165-6.

⁸ Kyburg, *op. cit.*, p. 168.

⁹ *Ibid.*, p. 171.

¹⁰ Helen Beebe and David Papineau, "Probability as a Guide to Life," *The Journal of Philosophy* 94 (1997): 217-243.

¹¹ *Ibid.*, p. 217.

¹² *Ibid.*, p. 219.

¹³ *Ibid.*, p. 223.

¹⁴ Ibid., p. 224.

¹⁵ Ibid, p. 226.

¹⁶ Elizabeth Kolbert, "Crash Course: Can a seventeen-mile-long collider unlock the universe?", *The New Yorker*, May 14, 2007. This article can be found online at http://www.newyorker.com/reporting/2007/05/14/070514fa_fact_kolbert.

¹⁷ Ibid.