

Rules so far

Probability of A Union $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability $P(A | B) = \frac{P(A \cap B)}{P(B)}$

Probability of an Intersection $P(A \cap B) = P(B)P(A | B)$

The following is some data from an experiment in smoking succession . Two groups of smokers were involved in the experiment (120 in each group). One group was given a Nicotine Patch and the second was given a similar device which was actually a placebo. All subjects agreed to try to quit smoking and after 8 weeks they were asked if they were still smoking.

1. **Let Event A = receive the nicotine patch. What is the probability Event A (nicotine patch)?**

SAS Output

SMOKING TREATMENT EXPERIMENT				
Treatment	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Nicotine	120	50.0	120	50.0
Placebo	120	50.0	240	100.0

$P(A) =$ _____

2. **Let Event B = No Longer Smoking. What is the probability Event B (No)?**

SAS Output

STILL SMOKING AFTER 8 WEEKS

SMOKING	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Yes	160	66.7	160	66.7
No	80	33.3	240	100.0

$P(B) =$ _____

Probability When Using Contingency Tables

3. What is the union between events A and B? That is being both receiving a Nicotine Patch and No Longer Smoking? $(A \cup B) =$

We have a cross-tabulation of Treatment by Still Smoking after 8 weeks

	Yes	No	Row Total
Nicotine Patch	64	56	120
Placebo	96	24	120
Column Total	160	80	240

_____ Everyone who received the Nicotine Patch
 _____ + Everyone who is No Longer Smoking
 _____ - Everyone who received the Nicotine Patch and is No Longer Smoking
 _____ $(A \cup B)$

And the Probability of Events A and B $P(A \cup B) =$ _____

4. What about the Intersection of receiving the patch and no longer smoking?
 $(A \cap B)$

_____ Everyone who both received the patch and is no longer smoking

What is the $P(A \cap B)$? _____

5. Conditional Probability is stated as “*The Probability of No Longer Smoking given you received the Nicotine Patch*”, and it is defined as:

$$P(B|A) = P(B \cap A)/P(A)$$

$P(B|A) =$ _____

6. The Complement of A, expressed as A^c would be Received the Placebo.

What is the $P(A^c) =$ _____

What is $P(A^c \cap B) =$ _____

STAT 200

Probability When Using Contingency Tables

7. So what is the Probability of No Longer Smoking given you received the placebo?

$$P(B|A^c) = P(B \cap A^c)/P(A^c)$$

$$P(B|A^c) = \underline{\hspace{15em}}$$

8. Look at the Table from the SAS program:

TABLE OF TREATMENT BY STILL SMOKING

TREATMENT	STILL SMOKING		
Frequency,			
Percent ,			
Row Pct ,			
Col Pct ,	YES	NO	Total

NICOTINE ,	64 ,	56 ,	120
	, 26.67 ,	, 23.33 ,	, 50.00
	, 53.33 ,	, 46.67 ,	
	, 40.00 ,	, 70.00 ,	

PLACEBO ,	96 ,	24 ,	120
	, 40.00 ,	, 10.00 ,	, 50.00
	, 80.00 ,	, 20.00 ,	
	, 60.00 ,	, 30.00 ,	

Total	160	80	240
	66.67	33.33	100.00

You have three different percentages for the first cell (Nicotine Patch and Still Smoking):

Percent	The cell value over the total	26.67 = 64/240 * 100
Row Pct	The cell value over the row margin on the right	53.33 = 64/120 * 100
Col Pct	The cell value over the column margin on the bottom	40.00 = 64/160 * 100

Note in the second cell that:

$P(A \cap B) = .2333$	is the percent
$P(B A) = P(A \cap B)/P(A) = .467$	is the row proportion for those No Longer Smoking who received the Nicotine Patch
$P(A B) = P(A \cap B)/P(B) = .700$	is the column proportion for those No Longer Smoking who received the Nicotine Patch

STAT 200
Probability When Using Contingency Tables

9. So what do you think the $P(A|B) = P(A \cap B)/P(B) =$ _____

This is the Probability of Receiving the Placebo Patch given you are No Longer Smoking.
 Does this make any sense?

Notes about Percentaging a Table and Independence:

10. Be careful how you look at data, and how you percentage it!

- The leading cause of death of children 1 to 4 years of age is accidents!
- 36% of all deaths for this age group comes from accidents
- One might conclude there is an accident waiting to happen for our children

- But the death rate for children 1 to 4 is 38.3 per 100,000 children
- This means the probability of a child aged 1 to 4 dying is only .000383
- And the overall probability of dying from an accident is .000138
- Anyone with toddlers knows that they are amazingly indestructible!
- But we also know how tragic it is when one dies from an accident

Cause of Death	Deaths	Nondeaths	Total
Accidents	2,147	?	?
All Others	3,801	?	?
Total	5,948	15,535,974	15,530,026

STAT 200
Probability When Using Contingency Tables

11. Next we will look at the odds and odds ratios.

The odds of the Nicotine Patch group Not Smoking versus smoking:

$$(\#No)/(\#Yes) = \underline{\hspace{4cm}}$$

$$(P(No)/P(Yes)) = \underline{\hspace{4cm}}$$

The odds of the Nicotine Patch group Smoking versus Not Smoking:

$$(\#Yes)/(\#No) = \underline{\hspace{4cm}}$$

$$(P(Yes)/P(No)) = \underline{\hspace{4cm}}$$

Note that this odds is the reciprocal of the first.

An odds Ratio is the ratio of two odds and compares the two odds, most often for two groups. We might want to compare the Odds Ratio of the Odds for Not Smoking versus Smoking for the Nicotine Patch versus the Placebo group.

Odds of Nicotine group Not Smoking versus Smoking = $\underline{\hspace{4cm}}$

Odds of Placebo group Not Smoking versus Smoking = $\underline{\hspace{4cm}}$

Odds Ratio = $\underline{\hspace{4cm}}$

What does this mean?

Probability When Using Contingency Tables

12. Extra Problem

- You are a doctor and a patient comes with a lump in her breast
- You know that there is only a 1% chance that it is malignant
 - Only 1,000 of 100,000 such lumps would be malignant
- But you urge a mamogram, or which you know they are:
 - 80% accurate for malignant lumps
 - 90% accurate for benign lumps
- The test comes back indicating that the lump is malignant
- **What is the probability that the lump is truly malignant, given the test indicates it is malignant?**
- **Hint:** build a mock table based on 100,000 people

Reality	Test Shows Malignant	Test Shows Benign	Total
Malignant			1,000
Benign			
Total			100,000

What is the probability that the lump is truly malignant, given the test indicates it is malignant?

Odds of a positive test for those with a malignant tumor _____

Odds of a positive test for those with a benign tumor _____

Odds Ratio = _____

Describe it in words: