

STAT 200 EXERCISE 5

1. **Alpha is the Greek term α .** In Inferential Statistics alpha represents the probability of making an error when we make an inference from a sample to the population. In most cases α is the probability of finding a sample with our z-score value or beyond - i.e., the area in the tail of the distribution, either positive or negative. In a confidence Interval, we divide α by 2 to spread this error in both tails.

We need to calculate the values of α for different Confidence Intervals. Fill in the following table. It will require you to find the z value that is associated with an $\alpha/2$ level of probability in the Standard Normal Table in the book.

Hint: the standard normal table shows the probability up to a value, and $\alpha/2$ is the probability after the value. It is easiest to find in the table the probability of $.5 - \alpha/2$ and then read the z-value that corresponds to it. For example, for the first example for $\alpha/2 = .125$, find the probability for $.5 - .125 = .375$ (or something very close to it) and read the z-value of 1.15 that corresponds to this probability.

Confidence Level	α	$\alpha/2$	Z $\alpha/2$
$100(1 - \alpha)$			
75%	.25	.125	1.15
80%	.20		
90%	.10		
95%	.05		
99%	.01		

2. **An experiment was conducted at MIT on the effect of melatonin on inducing sleep.** Young male volunteers were either given melatonin or a placebo. They were then placed in a dark room at midday and told to close their eyes for 30 minutes. The length of time it took them to fall asleep was recorded.

With the placebo the researchers found it took on average 15 minutes to fall asleep with a standard deviation of 5. We will assume this is the population parameters for young males.

That is, $\mu = 15$ and $\sigma = 5$

The data here are a random sample of young men who were given melatonin. For this data:

Minutes to Sleep for the Sample	
1	5 6 7 8
2	
3	1 2 3 4 9
4	0 4 4 5 5 7 8 8 9
5	0 0 1 1 9
6	0 0 1 1 1 2 2 3 3 4 4
7	5 6
8	2 3
9	
10	
11	
12	
13	
14	
15	6
16	2
Stem = whole number Leaf = decimal place	
Sum X = 222.1	n = 40
Sum X ² = 1570.43	

a. Calculate Descriptive statistics of the data

- Mean _____
- Median _____
- Mode _____
- Variance _____
- Standard Deviation _____
- Maximum _____
- Minimum _____
- Range _____

b. Calculate the **Standard Error** of the data:

c. Construct a **95% Confidence Interval** for this data. Use the Z-value you calculated from problem 1.

d. Calculate the Z-score for this sample mean as if it were part of a sampling distribution with $\mu = 15$ and $\sigma_x = 5/\sqrt{40}$ that is, as if the sample was from a population that was similar to the placebo group.

e. What is the probability of finding a sample mean equal to or less than the value you calculated if the population parameter for the mean is really 15. Hint: Use the z-score you calculated in d above, then look up this z-value in the table to find a probability. Did the melatonin seem to work?

