

B



## Rules so far

**Probability of  
A Union**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**Conditional  
Probability**

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

**Probability of  
an Intersection**

$$P(A \cap B) = P(B)P(A | B)$$

This is the “experiment” The following is some data from an experiment in smoking succession . Two groups of smokers were involved in the experiment (120 in each group). One group was given a Nicotine Patch and the second was given a similar device which was actually a placebo. All subjects agreed to try to quit smoking and after 8 weeks they were asked if they were still smoking.

1. **Let Event A = receive the nicotine patch. What is the probability Event A ( nicotine patch)?**

SAS Output

| SMOKING TREATMENT EXPERIMENT |           |         |                      |                    |
|------------------------------|-----------|---------|----------------------|--------------------|
| Treatment                    | Frequency | Percent | Cumulative Frequency | Cumulative Percent |
| Nicotine                     | 120       | 50.0    | 120                  | 50.0               |
| Placebo                      | 120       | 50.0    | 240                  | 100.0              |

P(A) = \_\_\_\_\_

2. **Let Event B = No Longer Smoking. What is the probability Event B ( No)?**

SAS Output

| STILL SMOKING AFTER 8 WEEKS |           |         |                      |                    |
|-----------------------------|-----------|---------|----------------------|--------------------|
| SMOKING                     | Frequency | Percent | Cumulative Frequency | Cumulative Percent |
| Yes                         | 160       | 66.7    | 160                  | 66.7               |
| No                          | 80        | 33.3    | 240                  | 100.0              |

P(B) = \_\_\_\_\_

Probability When Using Contingency Tables

3. What is the union between events A and B? That is being both receiving a Nicotine Patch and No Longer Smoking?  $(A \cup B) =$

We have a cross-tabulation of Treatment by Still Smoking after 8 weeks

|                | Yes | No | Row Total |
|----------------|-----|----|-----------|
| Nicotine Patch | 64  | 56 | 120       |
| Placebo        | 96  | 24 | 120       |
| Column Total   | 160 | 80 | 240       |

\_\_\_\_\_ Everyone who received the Nicotine Patch  
 \_\_\_\_\_ + Everyone who is No Longer Smoking  
 \_\_\_\_\_ - Everyone who received the Nicotine Patch and is No Longer Smoking  
 \_\_\_\_\_  $(A \cup B)$

And the Probability of Events A and B  $P(A \cup B) =$  \_\_\_\_\_

4. What about the Intersection of receiving the patch and no longer smoking?  
 $(A \cap B)$

\_\_\_\_\_ Everyone who both received the patch and is no longer smoking

What is the  $P(A \cap B)$ ? \_\_\_\_\_

5. Conditional Probability is stated as *“The Probability of No Longer Smoking given you received the Nicotine Patch”*, and it is defined as:

$$P(B|A) = P(B \cap A)/P(A)$$

$P(B|A) =$  \_\_\_\_\_

6. The Complement of A, expressed as  $A^c$  would be Received the Placebo.

What is the  $P(A^c) =$  \_\_\_\_\_

What is  $P(A^c \cap B) =$  \_\_\_\_\_

Probability When Using Contingency Tables

7. So what is the Probability of No Longer Smoking given you received the placebo?

$$P(B|A^c) = P(B \cap A^c)/P(A^c)$$

$$P(B|A^c) = \underline{\hspace{15em}}$$

8. Look at the Table from the SAS program:

TABLE OF TREATMENT BY STILL SMOKING

| TREATMENT  | STILL SMOKING |       | Total  |
|------------|---------------|-------|--------|
| Frequency, |               |       |        |
| Percent    |               |       |        |
| Row Pct    |               |       |        |
| Col Pct    | YES           | NO    |        |
| NICOTINE   | 64            | 56    | 120    |
|            | 26.67         | 23.33 | 50.00  |
|            | 53.33         | 46.67 |        |
|            | 40.00         | 70.00 |        |
| PLACEBO    | 96            | 24    | 120    |
|            | 40.00         | 10.00 | 50.00  |
|            | 80.00         | 20.00 |        |
|            | 60.00         | 30.00 |        |
| Total      | 160           | 80    | 240    |
|            | 66.67         | 33.33 | 100.00 |

You have three different percentages for the first cell (Nicotine Patch and Still Smoking):

|         |   |                        |
|---------|---|------------------------|
| Percent | The cell value over the total                       | $26.67 = 64/240 * 100$ |
| Row Pct | The cell value over the row margin on the right     | $53.33 = 64/120 * 100$ |
| Col Pct | The cell value over the column margin on the bottom | $40.00 = 64/160 * 100$ |

Note in the second cell that:

|                                    |  |
|------------------------------------|--|
| $P(A \cap B) = .2333$              | is the percent   |
| $P(B A) = P(A \cap B)/P(A) = .467$ | is the row proportion for those No Longer Smoking who received the Nicotine Patch    |
| $P(A B) = P(A \cap B)/P(B) = .700$ | is the column proportion for those No Longer Smoking who received the Nicotine Patch |

Probability When Using Contingency Tables

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9. So what do you think the  $P(A|B) = P(A \cap B)/P(B) =$  \_\_\_\_\_

This is the Probability of Receiving the Placebo Patch given you are No Longer Smoking. Does this make any sense?

**Notes about How to Percentage a Table and Independence:**

**10. Be careful how you look at data, and how you percentage it!**

- The leading cause of death of children 1 to 4 years of age is accidents!
- 36% of all deaths for this age group comes from accidents
- One might conclude there is an accident waiting to happen for our children
  
- But the death rate for children 1 to 4 is 38.3 per 100,000 children
- This means the probability of a child aged 1 to 4 dying is only .000383
- And the overall probability of dying from an accident is .000138
- Anyone with toddlers knows that they are amazingly indestructible!
- But we also know how tragic it is when one dies from an accident

| Cause of Death | Deaths | Nondeaths  | Total      |
|----------------|--------|------------|------------|
| Accidents      | 2,147  | ?          | ?          |
| All Others     | 3,801  | ?          | ?          |
| Total          | 5,948  | 15,535,974 | 15,530,026 |

Probability When Using Contingency Tables

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11. Next we will look at the odds and odds ratios.

The odds of the Nicotine Patch group Not Smoking versus smoking:

$$(\#No)/(\#Yes) = \underline{\hspace{4cm}}$$

$$(P(No)/P(Yes)) = \underline{\hspace{4cm}}$$

The odds of the Nicotine Patch group Smoking versus Not Smoking:

$$(\#Yes)/(\#No) = \underline{\hspace{4cm}}$$

$$(P(Yes)/P(No)) = \underline{\hspace{4cm}}$$

Note that this odds is the reciprocal of the first.

An odds Ratio is the ratio of two odds and compares the two odds, most often for two groups. We might want to compare the Odds Ratio of the Odds for Not Smoking versus Smoking for the Nicotine Patch versus the Placebo group.

$$\text{Odds of Nicotine group Not Smoking versus Smoking} = \underline{\hspace{4cm}}$$

$$\text{Odds of Placebo group Not Smoking versus Smoking} = \underline{\hspace{4cm}}$$

$$\text{Odds Ratio} = \underline{\hspace{4cm}}$$

What does this mean?

Probability When Using Contingency Tables

12. Extra Problem

- You are a doctor and a patient comes with a lump in her breast
- You know that there is only a 1% chance that it is malignant
  - Only 1,000 of 100,000 such lumps would be malignant
- But you urge a mamogram, or which you know they are:
  - 80% accurate for malignant lumps
  - 90% accurate for benign lumps
  
- The test comes back indicating that the lump is malignant
- **What is the probability that the lump is truly malignant, given the test indicates it is malignant?**
- **Hint:** build a mock table based on 100,000 people

| Reality   | Test Shows Malignant | Test Shows Benign | Total   |
|-----------|----------------------|-------------------|---------|
| Malignant |                      |                   | 1,000   |
| Benign    |                      |                   |         |
| Total     |                      |                   | 100,000 |

**What is the probability that the lump is truly malignant, given the test indicates it is malignant?**

\_\_\_\_\_

Odds of a positive test for those with a malignant tumor \_\_\_\_\_

Odds of a positive test for those with a benign tumor \_\_\_\_\_

Odds Ratio = \_\_\_\_\_

Describe it in words: