Standard Normal Curve Probability Distribution

The table is based on the upper right 1/2 of the Normal Distribution; total area shown is .5
The Z-score values are represented by the column value + row value, up to two decimal places
The probabilities up to the Z-score are in the cells

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Rejection regions for Common Values of Alpha

	Alternative Hypothesis		
	Lower Tailed	**Upper Tailed**	**Two Tailed**
alpha = .10	z < -1.28	z > 1.28	z < -1.645 or z > 1.645
alpha = .05	z < -1.645	z > 1.645	z < -1.96 or z > 1.96
alpha = .01	z < -2.33	z > 2.33	z < -2.575 or z > 2.575

NORMAL DISTRIBUTION

One bell-shaped, symmetrical distribution is the normal distribution. The normal distribution is perfectly symmetrical around the mean. It is also characterized by the fact that the mean, median, and mode of data that are normally distributed are all equal to each other.

The exact mathematical formula for the normal distribution is given below. It shows that the normal distribution is defined by two parameters which influence the center and spread of the distribution. These are:

μ the mean, which defines the center of the distribution

σ the standard deviation, which defines the spread of the data

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2}$$

Formula for the probability density distribution of the normal distribution

For every distribution with a mean (μ) and a standard deviation (σ) there is a different normal curve. Thus, there are an infinite number of normal curves. If X (a variable) is distributed as a normal variable then it is designated as:

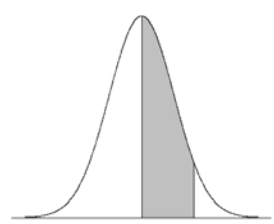
X ~ N(μ, σ) where values are given for μ and σ

Graphic of a Normal Distribution

The Standard Normal Table

Since its properties are defined by a formula, we can a priori define probabilities associated with the normal curve. If we convert our normally distributed variable to a z-score, we make it possible to use one table of probabilities for all normally distributed variables. This is called the Standard Normal Distribution table. Recall that with a z-score transformation of a variable, the Mean (μ) = 0 and the standard deviation (σ) = 1. We can use this information to make one table of probabilities based on the z-transformation. If you wish to use this table to determine probabilities for the normal distribution, you need to convert you data (or particular values of your data) to z-scores.

The standard normal probability table presented in most books (and on the reverse side of this sheet) on gives ½ of the area of the curve, since the distribution is symmetrical. Each half of the curve represents a probability of .5 (p = .5). The table allows you to find the probability from the center (i.e., the mean) to a z-score you are interested in. The table allows for up to two decimal places for the z-score, with the vertical axis showing the ones and first decimal place and the horizontal axis showing the second decimal place. The entries inside the table are the probabilities from the center (μ=0) up to the z-value you calculate. The shaded area in the graph to the right shows the probability from the center to your z-value, and this probability is what is read from the standard normal table.



The probability from the mean up to the z-value

Example 1. Suppose X is a normally distributed random variable with μ=10 and σ=2. Find the probability that X is greater than 7.34 and less than 12.66.

P(7.34 < X < 12.66)

We will use the following steps to solve for this.

1) Solve for z-scores using this formula.

$$Z = \frac{(X - \mu)}{\sigma}$$

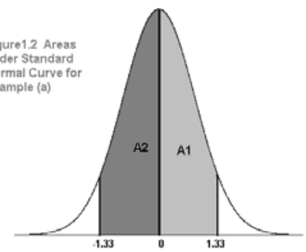
We were given μ = 10 and σ = 2, then by formula, the corresponding boundary Z values are:

$$Z = \frac{(7.34 - 10)}{2} = -1.33 \ \ and \ \ Z = \frac{(12.34 - 10)}{2} = 1.33$$

2) Sketch the standard normal distribution and shade the area corresponding to the probability you want to find.

Drawing out the area for which you want to find a probability is a useful strategy to keep you on task. We want to find the probabilities that z falls between -1.33 and 1.33, which is equivalent to the area between -1.33 and 1.33, showed highlighted in the figure to the right.

Figure1.2 Areas under Standard Normal Curve for Example (a)



3) Look up the probabilities in the Standard Normal Table

The Standard Normal Table provides the area between Z=0 and any non-negative value of Z. If we look up Z = 1.33 (the value in the 1.3 row and 0.03 column), we find that the area between Z=0 and Z=1.33 is 0.4082. This is the area labeled A1 in the figure. To find the area A2 located between Z=0 and Z=-1.33, we note that the symmetry of the normal distribution implies that the area between Z=0 and any point to the left is equal to the area between Z=0 and the point equidistant to the right. Thus, in this example the area between Z=0 and Z=-1.33 is equal to the area between Z=0 and Z=1.33. That is, A1 = A2 = 0.4082.

4) Do any final calculations

Finally, we need to add the two probabilities for A1 and A2 to find the answer. As a result:

P(7.34 < X < 12.66) = .4082 + .4082 = .8164

Example 2. Suppose we want to find the probability that X is greater than 12.9.

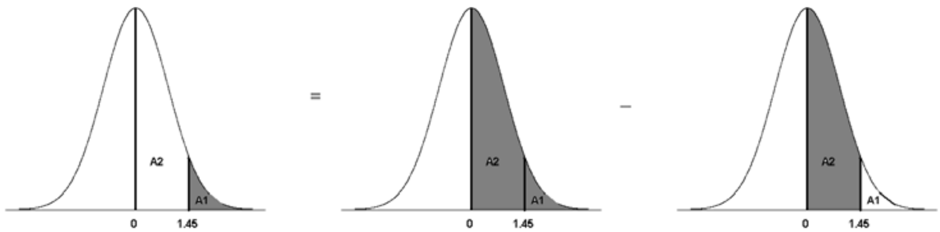
P(X > 12.9)

1) Solve for Z-scores

$$Z = \frac{(12.9 - 10)}{2} = 1.45$$

2) Sketch the standard normal distribution and shade the area corresponding to the probability you want to find.

In this case we are looking for the area beyond 12.9, or a z-value of 1.45. Since the standard normal table shows the probabilities up to the z-score you calculate, we need to subtract this probability from the probability of the entire ½ of the curve, or p=.5, in order to find the probability beyond that point. The following graphs illustrate this point.



3) Look up the probabilities in the Standard Normal Table

We look up the table to find the probability from 0 and Z corresponds to area A2 above. A2= 0.4265. However, this is not the final answer, because we want the probability beyond that point, contained in the area A1.

4) Do any final calculations

The final calculations involve subtracting from .5, the total area in the right hand side of the standard normal curve.

P (Z > 1.45) = 0.5 - 0.4265 = 0.0735

Summary: Steps for Finding a Probability Corresponding to a Normal Random Variable

1) **Solve for Z-scores**

2) **Sketch the standard normal distribution and shade the area corresponding to the probability you want to find.**

3) **Look up the probabilities in the Standard Normal Table**

4) **Do any final calculations**. If necessary, use the symmetry of the normal distribution to find areas corresponding to negative Z values and the fact that the total area on each side of the mean equals 0.5 to convert the areas from Normal table to the probabilities of the event you shaded.