FREC 408

Group Exercise 4

Discrete Random Variable Problem 4.27

- **A rock concert producer has scheduled an outdoor concert on a Saturday.**
- **If it does not rain, he expects to make $20,000 profit from the concert**
- **If it does rain, the producer will be forced to cancel the concert and lose $12,000 (from fees, advertising, stadium rental and so forth)**
- **The probability of rain on Saturday is .4**

- What is the expected profit from the concert?

Answer: Write out the probability distribution

x	**$20,000**	**-$12,000**
P(x)	**.6**	**.4**

E(x) = 20,000*.6 + -12,000*.4 = $7,200

- **For a fee of $1,000 an insurance company will insure against all losses from a rained out concert. If the producer buys the insurance, what is her expected profit from the concert?**

Answer

x	$20,000	0
P(x)	.6	.4

E(x) = (20,000*.6 + 0*.4) – 1,000 = $11,000

Assuming the forecast is accurate, do you believe the insurance company has charged too much or too little?

- **Let .6 be the probability they will not pay out**
- **Let .4 be the probability they will pay out**

x	$0	$12,000
P(x)	.6	.4

E(payment) = (0*.6 + 12,000*.4) = $4,800

They expect to pay out $4,800, but they only charged $1,000 – they charged too little!

The following table contains the probabilities associated with the number of accidents per day in a small sized city

#of Accidents per day (X)	0	1	2	3	4	5
P(x)	.10	.20	.45	.15	.05	.05

- Compute the expected number of accidents per day

The expected value of accidents is:

E(x) = 0(.10) + 1(.20) + 2(.45) + 3(.15) + 4(.05) + 5(.05)
= 2

The following table contains the probabilities associated with the number of accidents per day in a small sized city

#of Accidents per day (X)	0	1	2	3	4	5
P(x)	.10	.20	.45	.15	.05	.05

The variance and standard deviation are:

$\sigma^2 = (0-2)^2(.10) + (1-2)^2(.20) + (2-2)^2(.45) + (3-2)^2(.15) + (4-2)^2(.05) + (5-2)^2(.05)$
$= 1.4$
$\sigma = 1.183$

The following table contains the probabilities associated with the number of accidents per day in a small sized city

#of Accidents per day (X)	0	1	2	3	4	5
P(x)	.10	.20	.45	.15	.05	.05

What is the probability that there will be more than two accidents in a single day?

$p(x>2) = p(3) + p(4) + p(5) = .15 + .05 + .05 = .25$

The following table contains the probabilities associated with the number of accidents per day in a small sized city

#of Accidents per day (X)	0	1	2	3	4	5
P(x)	.10	.20	.45	.15	.05	.05

What is the probability that exactly 4 accidents will occur in a single day?

$p(x=4) = .05$

Let's revisit the psychic problem

- **Remember that a crystal is randomly placed under one of ten boxes and the psychic is asked to guess where it is.**
- **This experiment is repeated seven times, and x is the number of correct decisions in seven tries. Thus it is a Binomial variable.**
- **If the psychic is guessing, what is the value of p, the probability of a correct decision on each trial?**
  - p = .1
  - the probability of a "success" is .1

Let's view this as a discrete random variable – a binomial random variable

X	0	1	2	3	4	5	6	7
p(x)	**.4783**					**.0002**	**.0000**	**.0000**

Can you fill in the rest of the table?

To solve

- Use the table on page 800
  - **n = 7**
  - **p = .1**
  - **k = the values of our discrete random variable**
- **For p(x=1)**
  - **For k = 1, the probability is .850 which is the cumulative probability up and including 1**
  - **To find the exact p(x=1), subtract the value for k=0 from the value k = 1**
  - p(x=1) = .850 - .478 = .372

To solve

- Use the formula

$$p(1) = \frac{7!}{1!(7-1)!}(.1)^1(.9)^{7-1}$$

$$= \frac{5040}{720}(.0531) = .3720$$

Solve for all

X	0	1	2	3	4	5	6	7
p(x)	**.4783**	**.3720**	**.1240**	**.0230**	**.0026**	**.0002**	**.0000**	**.0000**

Can you solve for the mean and standard deviation of this binomial random variable?

Solve for Expected Value and Variance

X	0	1	2	3	4	5	6	7
p(x)	**.4783**	**.3720**	**.1240**	**.0230**	**.0026**	**.0002**	**.0000**	**.0000**

Expected value = mean = n•p = 7•.1 = .7

Variance = n•p•q = 7•(.1)•(.9) = .63

Standard Deviation = $(.63)^{.5}$ = .794

- **Probability of guessing wrong all 7 times for a random guess (p=.1) is**
  - **$.9^7 = .4783$**
  - **Or read it from the table!**

- **Probability of guessing wrong all seven times if you are a psychic with p =.5**
  - **$.5^7 = .0078$**
  - **Or read it from the Binomial table using n=7 and p=.5**

If a single bit of data (0 or 1) is transmitted over a noisy communication channel,

- It has a probability p of being incorrectly transmitted. To improve the reliability of the transmission, the bit is transmitted ***n*** times, where n is odd.
- A decoder at the receiving end, called a majority decoder, decides that the correct message is the one carried by the majority of the received bits.
- This means that if there are five transmissions of a (0,1) bit, **the bit used by at least three** of the transmissions would be considered correct.

Transmission Problem

- Assume that each bit is independently subject to being corrupted with the same probability p, and that p=.1. Note, p is the probability of an error, and in terms of this binomial problem
- we will think of X as the number of errors in n transmissions.

- If a company sent only one transmission, what is the probability of it being received without an error?

  - **If p = .1 is the probability of an error in one transmission,**
  - **1.0 - .1 = .9 is the probability of a reception without an error**

Transmission

X						
P(x)						

Transmission

X	0	1	2	3	4	5
P(x)						

Transmission

X	0	1	2	3	4	5
P(x)	.5905	.3281	.0729	.0081	.0005	.0000

The probabilities can be gotten from:

1. **The formula for the binomial with n= 5, p=.1 and solving for X = 0, 1, 2, 3, 4, 5**
2. **Using the binomial table, remembering that is shows the cumulative probability and you need to subtract in order to get individual probabilities**
3. **Use Excel and solve using the bionomdist function**

Calculate the mean, variance, and standard deviation for this problem.

- **Mean = np = 5(.1) = .5**
- **σ^2 = npq = 5(.1)(.9) = .45**
- **$\sigma = (npq)^{.5}$ = .67**

If five messages are sent for each bit, the probability that the message is correctly received is the probability of two or fewer errors. Explain why would this be so?

- The company is using a strategy where the majority of the bits sent will be the number chosen to be right.
- For example, say the true bit is 0 and the company sends it 5 times. If 3 of the 5 bits received are zero, the strategy will choose zero and be right. The same is true if 4 of 5 are zero, or if 5 of 5 are zero.
- However, if 0, 1, or 2 of the transmissions result in a zero received at the other end, the strategy will guess wrong and say that one is the right number.

Can you explain why this logic is reasonable?

- The probability that the transmission has two or fewer errors represents the situation where the strategy of choosing the majority will work.
- This represents the situation of where the transmission of five signals generates two or fewer mistakes, so that a majority decision point will lead to a correct decision.

What is the probability that the message is correctly received in five transmissions? Did sending five transmissions improve the chances of sending the message correctly?

- **We need to find the $P(X \leq 2)$ Which is cumulative binomial probability up to and including 2. From the table we read:**
  - **N=5, p=.1 X=2 cumulative = .991**
- So the probability that the message is correctly received in five transmissions is .991. This is a vast improvement over .900 and would be acceptable to most companies.