Difference of Means and ANOVA Problems

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FREC 408

Accounting Firm Study

- **An accounting firm specializes in auditing the financial records of large firm**
- **It is interested in evaluating its fee structure,particularly in relation to charges by the size of the firm**
- **It takes a random sample of 10 companies from three size classes**
  - **Sales over $250 million**
  - **Sales of $100 to $250 million**
  - **Sales of Less than $100 million**

Stem and Leaf Plot

STEM	Leaf
0	20 52 55 75 76 80 80 88 92
1	00 00 10 25 32 41 50 50 50 60 86
2	00 00 30 33 50 75
3	25
4	75
5	
6	00
7	
8	00

Descriptive Statistics with 95% C.I.

	$250+	*$100-$250*	*< $100*	*Total*
Mean	335.50	129.50	106.00	190.33
Standard Error	70.84	18.03	18.02	30.87
Median	262.50	121.00	90.00	145.50
Mode	150	#N/A	#N/A	150
Standard Deviation	224.02	57.02	56.99	169.08
Sample Variance	50185.83	3251.17	3247.78	28587.06
Kurtosis	0.64	-0.48	-0.51	5.79
Skewness	1.16	0.57	0.41	2.30
Range	700	178	180	780
Minimum	100	55	20	20
Maximum	800	233	200	800
Sum	3355	1295	1060	5710
Count	10	10	10	30
Confidence Level(95.0%)	160.26	40.79	40.77	63.13

Things to note

- **The mean cost for the largest firms is much larger than the other two firm classes**
- **While there is a difference in the means for the lower two classes, when we look at the BOE for the 95% C.I. there is overlap**
- **The variances of the lower two firms is very similar – but both are considerably lower than that for the largest firms**

Difference of Means Test

- **Let's test to see if there is a significant difference between firms of sales between**
  - **$100 to $250 million**
  - **Less than $100 million**
- **Is this reasonable?**
  - **Ratio of variances is**
  - **3251.17/3247.78 = 1.001**

Decision Tree for Two Means

Target	Assumptions	Test Statistic
H_0: μ_1-μ_2=D	Independent random samples Large sample size (n_1, n_2>30)	z, using sample variance
	Independent random samples Small sample size Populations appr. normal **Equal variances**	t, using pooled variance S_p^2

POOLED ESTIMATE OF THE VARIANCE

- **The Our formula will be a weighted average of s_1 and s_2**

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1+n_2-2)}$$

$$s_p^2 = \frac{(10-1)3251.17 + (10-1)3247.78}{(10+10-2)}$$

$$s_p^2 = \frac{58490.55}{18} = 3249.475 \qquad s_p = 57.004$$

NOTE

- **Since the sample sizes were equal, we could have simply taken the average of the two variances**
- **(3251.17 + 3247.78)/2 = 3249.475**

Use the Pooled Estimate of the Variance to calculate the standard error

$$\hat{\sigma}_{(\bar{x}_1-\bar{x}_2)} = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\hat{\sigma}_{(\bar{x}_1-\bar{x}_2)} = 57.004\sqrt{\frac{1}{10}+\frac{1}{10}} = 25.493$$

Accounting Firm Problem

Null hypothesis	**H_0: (μ_1-μ_2) = 0**
Alternative	**H_a: (μ_1-μ_2) ≠ 0 two-tailed test**
Assumptions	**Small independent samples, approx normal, variances are equal**
Test Statistic	**t* = (129.50-106.00 - 0)/25.493**
Rejection Region	**$t_{.05/2,\ 18\ d.f.}$ = 2.101**
Calculation	**t* = .922**
Conclusion	**t* < -$t_{.05/2,\ 18\ d.f.}$** **.922 < 2.101** **We cannot reject H_0: (μ_1-μ_2) = 0**

EXCEL Output

t-Test: Two-Sample Assuming Equal Variances

	$100-$250	*< $100*
Mean	129.500	106.000
Variance	3251.167	3247.778
Observations	10	10
Pooled Variance	3249.472	
Hypothesized Mean Difference	0	
df	18	
t Stat	0.922	
P(T<=t) one-tail	0.184	
t Critical one-tail	1.734	
P(T<=t) two-tail	0.369	
t Critical two-tail	2.101	

Let's shift to ANOVA

- **ANOVA would allow us to test the difference of means across all classes of firm size**
- **We need to assume equal variances – is this reasonable?**

Review Degrees of freedom

- **k = 3**
- **n = 30**

- **SS(Total) df = n-1 = 30 - 1 = 29**
- **SST df = k-1 = 3 – 1 = 2**
- **SSE df = n-k = 30 – 3 = 27**

Excel ANOVA Output

Anova: Single Factor						
SUMMARY						
Groups	*Count*	*Sum*	*Average*	*Variance*		
$250+	10	3355	335.50	50185.83		
$100-$250	10	1295	129.50	3251.17		
< $100	10	1060	106.00	3247.78		
ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	318862	2	159430.83	8.44	0.00	3.35
Within Groups	510163	27	18894.93			
Total	829025	29				

ANOVA Hypothesis Test

- **Null hypothesis**
- **Alternative**
- **Assumptions**
- **Test Statistic**
- **Rejection Region**
- **Conclusion**

- **H_0: $\mu_1=\mu_2=\mu_3$**
- **H_a: At least two means differ**
- **Equal variances, normal distribution**
- **F* = 8.44**
- **$F_{.05,\ 2,\ 27\ d.f.}$ = 3.35**
- **F* > F**
- **8.44 > 3.35**
- **Reject H_0: $\mu_1=\mu_2=\mu_3$**

Brake Test

- A firm makes disc brakes for the automobile industry
- The R&D department tested four different brake systems
- In the test, they used 40 identical mid-sized cars, 10 each for the four brake systems
- The cars were driven on a test track and stopped electronically
- They measured the distance in feet to bring the car to a stop
- **Are there differences in stopping distance across the different brakes?**

The data

	Brake A	*Brake B*	*Brake C*	*Brake D*	*Total*
Mean	272.300	271.200	262.300	265.100	267.725
Standard Error	2.290	2.525	1.476	3.250	1.360
Median	274.500	270.000	264.000	259.000	267.500
Mode	275	267	264	259	257
Standard Deviation	7.243	7.983	4.668	10.279	8.602
Sample Variance	52.456	63.733	21.789	105.656	73.999
Kurtosis	0.192	0.270	-1.646	-0.641	-0.785
Skewness	-0.722	0.714	-0.008	0.945	0.399
Range	24	26	12	28	32
Minimum	259	261	257	255	255
Maximum	283	287	269	283	287
Sum	2723	2712	2623	2651	10709
Count	10	10	10	10	40
Confidence Level(95.0%)	5.181	5.711	3.339	7.353	2.751

The Output

Anova: Single Factor						
SUMMARY						
Groups	*Count*	*Sum*	*Average*	*Variance*		
Brake A	10	2723	272.300	52.456		
Brake B	10	2712	271.200	63.733		
Brake C	10	2623	262.300	21.789		
Brake D	10	2651	265.100	105.656		
ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	693.275	3	231.092	3.794	0.018	2.866
Within Groups	2192.700	36	60.908			
Total	2885.975	39				

ANOVA Hypothesis Test

- **Null hypothesis**
- **Alternative**
- **Assumptions**
- **Test Statistic**
- **Rejection Region**
- **Conclusion**

- **H_0: $\mu_1=\mu_2=\mu_3=\mu_4$**
- **H_a: At least two means differ**
- **Equal variances, normal distribution**
- **$F^* = 3.794$**
- **$F_{.05,\ 3,\ 36\ d.f.} = 2.866$**
- **$F^* > F$**
- **3.794 > 2.866**
- **Reject H_0: $\mu_1=\mu_2=\mu_3=\mu_4$**

What is R-square?

- **693.275/2885.975 = .24**

Cock Roach Data

On page 660 there is a Statistics in the Real World problem on roaches.

- This is a study to see if the navigation of cockroaches is random or not.
- The researcher hypothesized that cockroaches do follow trails, much like bees, ants, and termites, using chemical trails.
- She used a chemical trail with pheromones as the main treatment (EXTRACT - from cockroach feces), but also included a control trail using methanol (TRAIL).
- **Today, we will focus on a Two-Way ANOVA of the Trail Type (Extract versus Control group) and Roach Type (Gravid, Male, Female, Nymph)**

Roach Two-Way ANOVA

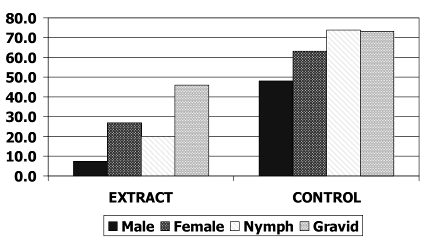
- The researcher released German cockroaches of different age, sex, and reproductive stage to see if these factors influenced trail following ability
  - **Factor** = TYPE:
    - **The levels = Female; Gravid; Male; and Nymph**
  - **Factor** = TRAIL
    - **The levels = Extract and Control**
- She measured the movement pattern of the cockroaches and calculated an average perpendicular distance
  - **Response Variable** = MOVE

	Extract	Control	Male	Female	Gravid	Nymph	Total
Mean	**25.18**	**64.54**	**27.73**	**45.05**	**59.62**	**47.03**	**44.86**
Standard Error	4.19	5.97	6.94	9.60	6.51	9.49	4.25
Median	13.55	54.45	10.10	31.00	54.45	29.90	37.25
Mode	2.40	#N/A	2.40	#N/A	#N/A	#N/A	2.40
Standard Deviation	26.53	37.75	31.04	42.95	29.12	42.44	37.99
Sample Variance	**703.67**	**1424.76**	**963.18**	**1844.44**	**848.24**	**1800.73**	**1442.95**
Kurtosis	0.76	-0.05	-0.33	1.76	-0.08	0.07	0.30
Skewness	1.33	0.67	1.08	1.42	0.00	1.11	0.93
Range	99.10	155.10	91.00	159.40	113.10	129.60	161.90
Minimum	2.10	8.90	2.10	4.60	4.80	3.30	2.10
Maximum	101.20	164.00	93.10	164.00	117.90	132.90	164.00
Sum	1007.00	2581.40	554.60	900.90	1192.30	940.60	3588.40
Count	40	40	20	20	20	20	80

The is how the ANOVA procedure arranges the Data

SUMMARY	Male	Female	Gravid	Nymphs	Total
Extract					
Count	10	10	10	10	40
Sum	74.60	269.00	460.50	202.90	1007.00
Average	**7.46**	**26.90**	**46.05**	**20.29**	**25.18**
Variance	90.94	1081.62	766.47	247.53	703.67
Control					
Count	10	10	10	10	40
Sum	480.00	631.90	731.80	737.70	2581.40
Average	**48.00**	**63.19**	**73.18**	**73.77**	**64.54**
Variance	1029.40	2080.55	615.34	1965.07	1424.76
Total					
Count	20	20	20	20	
Sum	554.6	900.9	1192.3	940.6	
Average	**27.73**	**45.045**	**59.615**	**47.03**	
Variance	963.182	1844.44	848.2371	1800.73	

I can graph it to try to make sense of it



R^2 = 14164/45082.5 = .314

ANOVA

Anova: Single Factor						
SUMMARY						
Groups	*Count*	*Sum*	*Average*	*Variance*		
Female	20	421.4	21.07	682.79		
Gravid	20	880.6	44.03	616.86		
Male	20	147.5	7.375	74.12		
Nymph	20	374.6	18.73	253.51		
ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	14164	**3**	4721.342	**11.605**	0.000002	4.050
Within Groups	**30918.5**	76	**406.822**			
Total	45082.5	79				

Conduct a Test to see if there is a mean difference in MOVE by the levels for Type (Male, Female, Nymph, and Gravid). Use an F-test with α= .01.

Null Hypothesis	$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
Alternative Hypothesis	H_a: At least one mean is different
Assumptions of Test	Large sample, equal variances, normal distribution
Test Statistic	$F^* = 11.605$
Rejection Region	$F_{.01,\,3 \text{ and } 76 \text{ d.f.}} = 4.05$
Comparison of Test Statistics with Rejection Region	$F^* > F_{.01,\,3 \text{ and } 76 \text{ d.f.}}$ 11.605 > 4.05 We can reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ p-value < .0001

What if I focused on just the TRAIL Factor?

Anova: Single Factor						
SUMMARY						
Groups	*Count*	*Sum*	*Average*	*Variance*		
Extract	40	1007.00	25.18	703.67		
Control	40	2581.40	64.54	1424.76		
ANOVA						
ource of Variatio	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Group	30984.19	1	30984.19	29.11	0.000	3.96
Within Groups	83008.85	78	1064.22			
Total	113993.04	79				

R^2 = 30,984.19/113,993.04 = .2718 or 27% of the variability in MOVE

What if I focused on just the TRAIL Factor?

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
Male	20	554.60	27.73	963.18
Female	20	900.90	45.05	1844.44
Gravid	20	1192.30	59.62	848.24
Nymph	20	940.60	47.03	1800.73

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	10317.80	3	3439.27	2.52	0.06	2.72
Within Groups	103675.24	76	1364.15			
Total	113993.04	79				

R^2 = 10,317.89/113,993.04 = .0905 or 9% of the variability in MOVE

Two Factor ANOVA

SUMMARY	Male	Female	Gravid	Nymphs	Total
Extract					
Count	10	10	10	10	40
Sum	74.60	269.00	460.50	202.90	1007.00
Average	**7.46**	**26.90**	**46.05**	**20.29**	**25.18**
Variance	90.94	1081.62	766.47	247.53	703.67
Control					
Count	10	10	10	10	40
Sum	480.00	631.90	731.80	737.70	2581.40
Average	**48.00**	**63.19**	**73.18**	**73.77**	**64.54**
Variance	1029.40	2080.55	615.34	1965.07	1424.76
Total					
Count	20	20	20	20	
Sum	554.6	900.9	1192.3	940.6	
Average	**27.73**	**45.045**	**59.615**	**47.03**	
Variance	963.182	1844.44	848.2371	1800.73	

Two-Factor ANOVA

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	30984.19	1	30984.19	31.47	0.00	3.97
Columns	10317.80	3	3439.27	3.49	0.02	2.73
Interaction	1798.82	3	599.61	0.61	0.61	2.73
Within	70892.22	72	984.61			
Total	113993.04	79				

Sample refers to TRAIL (Extract versus Control)

Columns refers to TYPE (Male, Female, Gravid, Nymph)

Interaction refers to the interaction between TRAIL and TYPE

How to Solve for R^2?

- **Use this formula: 1 – SSE/SST**
- **R^2 = 1 – 70,892.22/113,993.02**
  - **= 1 - .6219**
  - **= .3781**
  - **38% of the variability in MOVE is accounted for by considering the trail type and the Roach Type**