The Aggregate Expenditures Model And The Multiplier

Economics 152

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1 Introduction

In this handout, we will look at the Aggregate Expenditures Model or the Keynesian Model and several Multipliers. We will look at these multipliers within the context of the complete Keynesian model. In computing these multipliers we will assume that the government imposes a proportional income tax.[1] Finally, we will look at the Balanced Budget multiplier.

2 The Keynesian Multiplier

We can compute several multipliers associated with the Keynesian model. In particular, we can compute the government expenditures multiplier associated with an increase in government expenditures, the transfer payments multiplier associated with an increase in transfer payments and the taxes multiplier associated with either a lump-sum tax or a proportional income tax. In addition, one can also look at the investment multiplier. We will look at each of these cases. But before we do that, let us look at the model.

The equilibrium condition in this model is again Output = Aggregate Expenditures. Denoting output by Y and aggregate expenditures by AE, we have the following equilibrium condition:

$$Y = AE \quad (1)$$

[1] I will leave the computation of these multipliers in the case of lump-sum taxes to you as an exercise.

$$Y = C + I_g + G + X_n \tag{2}$$

We will assume that consumption is dependent on disposable income $C = \bar{C} + cY_D$ investment (I_g) government expenditures (G) and net exports (X_n) are *autonomous or exogenous.* These assumptions are represented as

$$\begin{aligned} C &= \bar{C} + cY_D && (3)\\ I_g &= \bar{I}_g && (4)\\ G &= \bar{G} && (5)\\ X_n &= \bar{X}_n && (6) \end{aligned}$$

Disposable income as before is defined as

$$Y_D = Y + TR - TA \tag{7}$$

2.1 An Increase In Government Expenditures

Assuming proportional income taxes (i.e. $TA = t\,Y$ with $0 < t < 1$), the government expenditures multiplier can be computed. This is shown below:

$$\begin{aligned} Y &= \bar{C} + cY_D + \bar{I}_g + \bar{G} + \bar{X}_n \\ Y &= \bar{C} + c(Y + \overline{TR} - t\,Y) + \bar{I}_g + \bar{G} + \bar{X}_n \\ \triangle Y &= (\triangle\bar{C} + c\,\triangle\,\overline{TR} + \triangle\bar{I}_g + \triangle\bar{G} + \triangle\bar{X}_n) + c(1-t)\,\triangle\,Y && (8) \end{aligned}$$

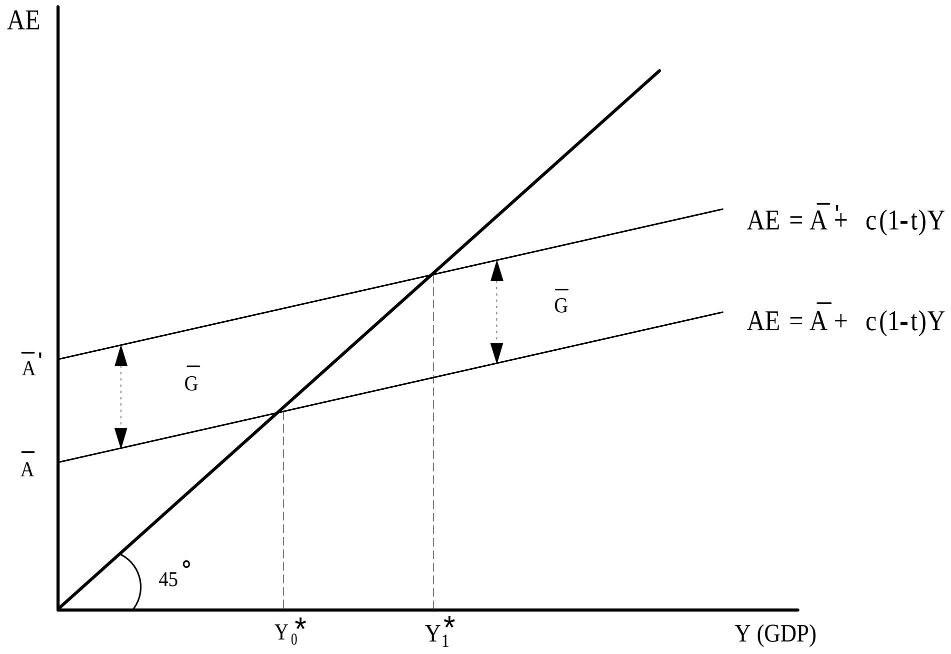
To compute the government expenditures multiplier, we set $\triangle\bar{C} = c\triangle\overline{TR} = \triangle\bar{I}_g = \triangle\bar{X}_n = 0$. Thus, we have the following:

$$\begin{aligned} \triangle Y &= \left(\frac{1}{1 - c(1-t)}\right)\triangle\,\bar{G} \\ \frac{\triangle Y}{\triangle\bar{G}} &= \left(\frac{1}{1 - c(1-t)}\right) && (9) \end{aligned}$$

Equation (9) represents the government expenditures multiplier.[2] Graphically, an increase in G using the Keynesian Cross diagram is shown on the next page.[3]

[2] Under the above assumptions, the investment multiplier is exactly identical to the government expenditures multiplier.

[3] The Keynesian Cross diagram for the case of an increase in I_g is also identical to that of an increase in G.



Keynesian Cross Diagram

2.2 An Increase In Transfer Payments

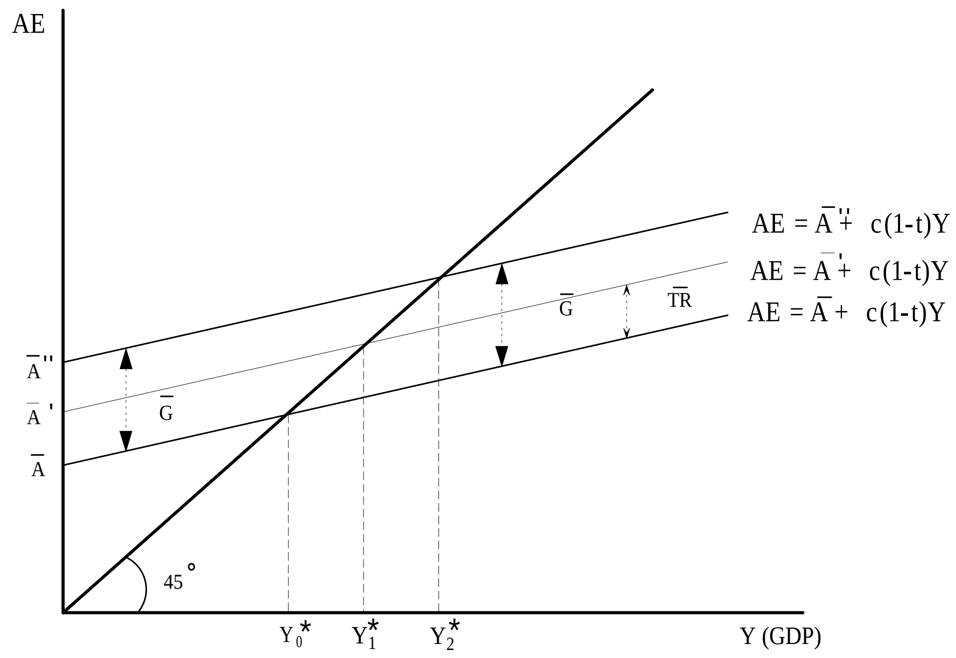
Assuming proportional income taxes, the transfer payments multiplier can be computed. This is shown below:

$$
\begin{aligned}
Y &= \bar{C} + cY_D + \bar{I}_g + \bar{G} + \bar{X}_n \\
Y &= \bar{C} + c(Y + \overline{TR} - t\,Y) + \bar{I}_g + \bar{G} + \bar{X}_n \\
\triangle Y &= (\triangle\bar{C} + c\,\triangle\,\overline{TR} + \triangle\bar{I}_g + \triangle\bar{G} + \triangle\bar{X}_n) + c(1-t)\,\triangle\,Y \qquad (10)
\end{aligned}
$$

To compute the transfer payments multiplier, we set $\triangle\bar{C} = \triangle\bar{I}_g = \triangle\bar{G} = \triangle\bar{X}_n = 0$. Thus, we have the following:

$$
\begin{aligned}
\triangle Y &= \left(\frac{1}{1-c(1-t)}\right) c\,\triangle\,\overline{TR} \\
\frac{\triangle Y}{\triangle\overline{TR}} &= \left(\frac{1}{1-c(1-t)}\right) \times c \qquad (11)
\end{aligned}
$$

Equation (11) represents the transfer payments multiplier. Graphically, an increase in transfer payments is shown below using the Keynesian Cross diagram.

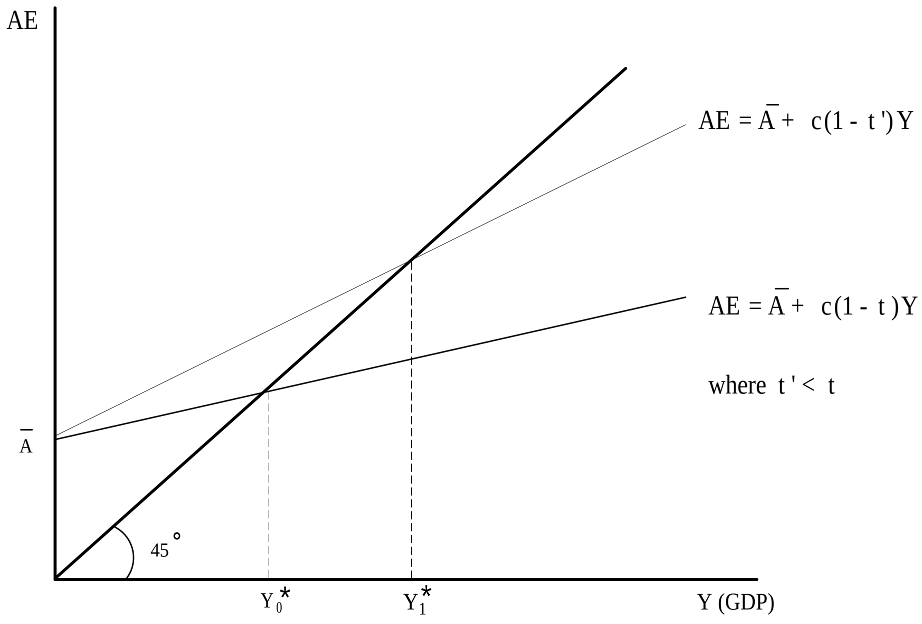


Keynesian Cross Diagram

Note that in the above Keynesian Cross Diagram, the increase in equilibrium GDP associated with an increase in G is greater than the increase in equilibrium GDP associated with an increase in TR. This is because a fraction of the increase in transfer payments is consumed while the rest is saved whereas the total amount of the increase in G is spent.

2.3 A Cut In Proportional Income Taxes

A cut in proportional income taxes is shown using the Keynesian Cross diagram below.[4]



Keynesian Cross Diagram

2.4 A Cut In Lump-Sum Taxes

Assuming a cut in lump-sum taxes (i.e. $TA = \overline{TA}$), the lump-sum taxes multiplier can be computed. This is shown below.

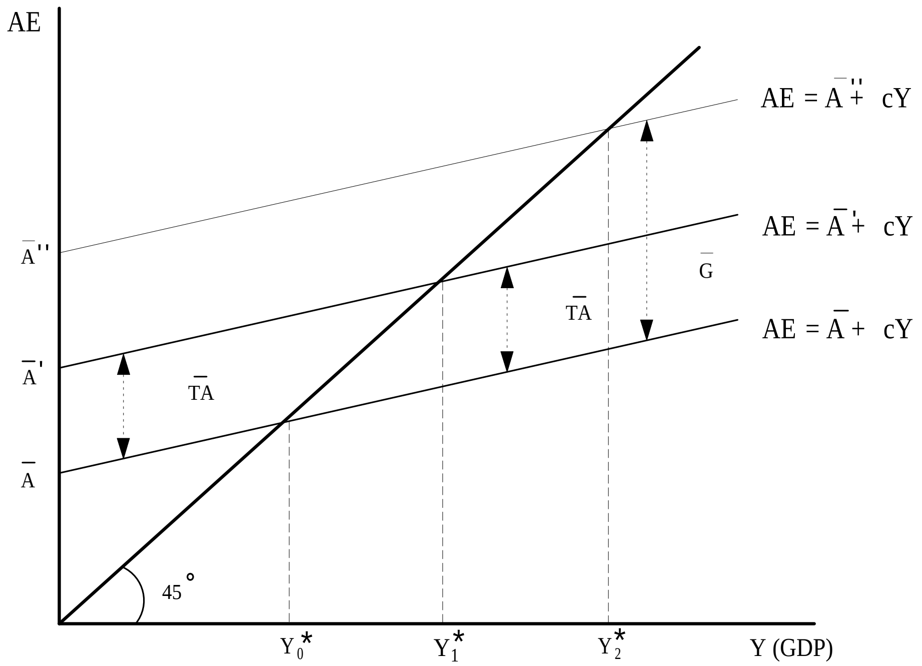
$$\begin{aligned} Y &= \bar{C} + cY_D + \bar{I}_g + \bar{G} + \bar{X}_n \\ Y &= \bar{C} + c\,(Y + \bar{TR} - \bar{TA}) + \bar{I}_g + \bar{G} + \bar{X}_n \\ Y &= (\bar{C} + c\bar{TR} - c\bar{TA} + \bar{I}_g + \bar{G} + \bar{X}_n) + cY \\ \triangle Y &= (\triangle\bar{C} + c \triangle \overline{TR} - c \triangle \bar{TA} + \triangle\bar{I}_g + \triangle\bar{G} + \triangle\bar{X}_n) + c \triangle Y \quad (12) \end{aligned}$$

[4] I am not going over the algebraic derivation of the proportional income taxes multiplier since that would require a little bit of Calculus.

To compute the lump-sum tax multiplier, we set $\triangle\bar{C} = c \triangle \overline{TR} = \triangle\bar{I}_g = \triangle\bar{G} = \triangle\bar{X}_n = 0$. Thus, we have the following:

$$
\begin{aligned}
\triangle Y &= \left(\frac{1}{1-c}\right) \times -c \triangle \overline{TA} \\
\frac{\triangle Y}{-\triangle \overline{TA}} &= \left(\frac{1}{1-c}\right) \times c
\end{aligned}
\tag{13}
$$

Equation (12) represents the lump-sum tax multiplier. Graphically, a cut in lump-sum taxes is shown below using the Keynesian Cross diagram.



Keynesian Cross Diagram

Note that in the Keynesian Cross Diagram shown above the increase in equilibrium GDP associated with a lump-sum tax cut is smaller than the increase in equilibrium GDP associated with an increase in G. This is because only a fraction of the tax cut is spent while the rest is saved, whereas the total amount of the increase in G is spent.

3 The Balanced Budget Multiplier

We will now look at the balanced budget multiplier (BBM). Before we do that let us define the budget surplus (BS) and its counterpart the budget deficit (BD). The budget surplus is defined as the excess of tax revenues over government expenditures. This is represented as

$$BS = TA - (G + TR)$$

The budget deficit is defined as a negative budget surplus. This is represented as

$$\begin{aligned} BD &= -BS \\ BD &= (G + TR) - TA \end{aligned}$$

We will look at the balanced budget multiplier under the following set of assumptions:

- the budget is balanced, i.e. revenues from taxes is exactly equal to government expenditures
- the government imposes a proportional income tax and spends the entire tax revenues as government expenditures on goods and services

We will see that given the above assumptions the balanced budget multiplier is equal to unity. This is formally shown below:

$$\begin{aligned} Y &= AE \\ Y &= \bar{A} + c(1-t)Y \\ Y &= \bar{A} + c(Y - tY) \\ Y &= \bar{A} + c(Y - TA) \\ \triangle Y &= \triangle\bar{G} + c(\triangle Y - \triangle TA) \\ (1-c)\triangle Y &= \triangle\bar{G} - c\triangle TA \\ \triangle Y &= \frac{1}{1-c}(\triangle\bar{G} - c\triangle TA) \end{aligned} \tag{14}$$

The government imposes a proportional income tax and spends the entire tax revenues as government expenditures on goods and services. This assumption is represented as

$$\triangle TA = \triangle\bar{G} \tag{15}$$

Substituting equation (15) in equation (14) we have the following

$$\begin{aligned}
\triangle Y &= \frac{1}{1-c}(\triangle\bar{G} - c\triangle\bar{G}) \\
\triangle Y &= \left(\frac{1}{1-c}\right)(1-c)\triangle\bar{G} \\
\triangle Y &= \triangle\bar{G} \qquad (16)
\end{aligned}$$

Given equation (15) we have the following

$$\triangle Y = \triangle\bar{G} = \triangle TA \qquad (17)$$

Thus the BBM is

$$\frac{\triangle Y}{\triangle\bar{G}} = 1 = \frac{\triangle Y}{\triangle TA} \qquad (18)$$

Equation (18) says that a change in output associated with an increase in G is the same as the change output associated with an increase in TA under the assumptions of the BBM.