Name _____ Login name _____

General Instructions

- Only write your name on the *first* page of this document, in the blank provided.
- Turn off any noise making device, especially **CELL PHONES**. You may lose up to one letter grade if your device disturbs the peace of the exam.
- Epsilon transitions are accepted in answers to this exam unless a question explicitly says they are not.
- You have 75 minutes. **Pace yourself,** and pay attention to the point values.
- I am here to answer questions. That said, **Read questions three times before asking a question.** If you still have a question, raise your hand and wait to be acknowledged by the professor. Ask questions *quietly.*
- Do problems you are confident about first. If you finish the problems you know, write what you do know about other problems to gain partial credit; but erroneous information may detract from that credit or irritate the grader, so don't make stuff up.
- Read *all* the directions *carefully* on each problem. The Red Knot, an endangered bird, migrates over 10,000 miles every Spring to arrive hungry at the Delaware Bay just as the horseshoe crabs lay their eggs. Delaware fishermen cut up millions of horseshoe crabs for cheap bait, and now the toll on the horseshoe crab population may doom the Red Knot.
- Often writing a fast, rough version of a proof/machine in English or drawing will make your answering faster and more accurate. It may enable me to give partial credit in some circumstances.
- Have fun!

For the following problems, informally outline the way you would approach the proof described, as we did in class.

1. (8 pts) You are given two regular languages, L_1 and L_2. How could you use epsilon transitions to prove constructively that $L_1 \cup L_2$ is also regular? Draw your informal approach (as discussed in class), labeling all parts of your drawing[1].

2. (8 pts) You are given a regular language, L_1. How could you use epsilon transitions to prove constructively that L_1^* is also regular? Draw your informal approach (as discussed in class), labeling all parts of your drawing.

3. (8 pts) You are given two regular languages, L_1 and L_2. How could you use epsilon transitions to prove constructively that $L_1 L_2$ (concatenation) is also regular? Draw your informal approach (as discussed in class), labeling all parts of your drawing.

[1] For example, if there is an egg shape in your drawing, tell me what it represents, not that it is an egg.

4. Bootsy asks Flea to write a CFG for the language

$$A = \{ww \mid w \in \{0,1\}^*\}$$

and Flea returns the grammar G = ($\{$S$\}$, $\{$0,1$\}$, P, S) , where P is the following set of productions:

$$\text{S} \longrightarrow 0\text{S}0\text{S} \mid 1\text{S}1\text{S} \mid \epsilon$$

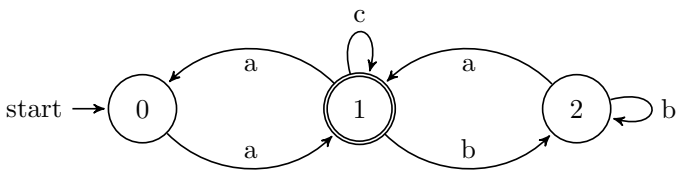
(a) (8 pts) Use G to draw a full parse tree for the string 010010.

(b) (6 pts) Is L(G) a correct grammar for the language Bootsy specified? Is yes, explain why. If not, give a counter-example **and** parse tree.

(c) (6 pts) Is L(G) a CFL? Is yes, explain why. If not, give a counter-example and explain why it is a counter example.

5. (10 pts) Draw a transition diagram for an NDFA[2] that accepts only the set
   A = $\{x \in \{a,b\}^* \mid \#a(x)$ is divisible by two or three $\}$

6. (20 pts) Consider the machine shown. For each pattern below, construct a regular expression inductively as described in class and presented on pages 51-53 of Kozen. Please show your work — you will not get credit if you do not break each expression down at least one level before writing the regexp.



$\alpha_{11}^{\{2\}} =$

$\alpha_{11}^{\{0,2\}} =$

$\alpha_{02}^{\{0,2\}} =$

[2]Epsilon is acceptable.

7. (20 pts) Write a formal definition of an NPDA that recognizes the set of strings L:

$$L = \{a^k b^m c^n \mid k,m \geq 0 \text{ and } n=2*m\}$$

Acceptance should be by accept state.